



## From quantum gates to weak traces & annealers

Ajmeri (Shushmi) Chowdhury Computational Nonlinear & Quantum Optics Group, Department of Physics, University of Strathclyde











## **Introduction to Quantum Computing** 2020-21

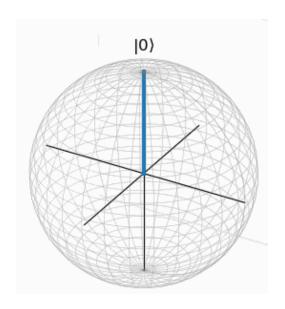




Qiskit

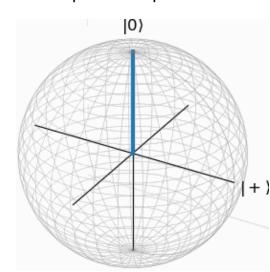
#### X gate

180° around the X axis



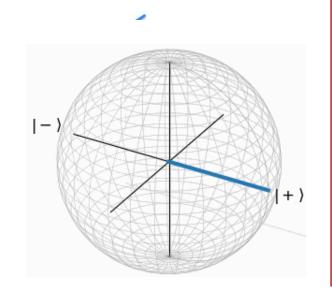
#### **Hadamard gate**

Creates equal superposition of |0> and |1>

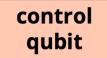


#### **Z** gate

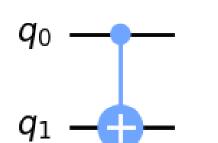
180° around the Z axis



#### The CX Gate







#### **Quantum Key Distribution**



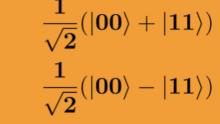




**Explained: BB84 & E91 Prtocols** 



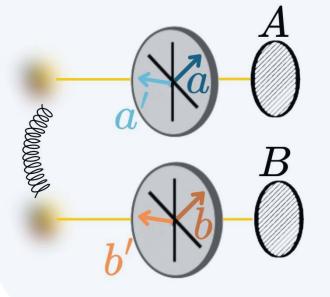
#### **Bell States (EPR Pairs)**



$$rac{1}{\sqrt{2}}(\ket{01}-\ket{10})$$

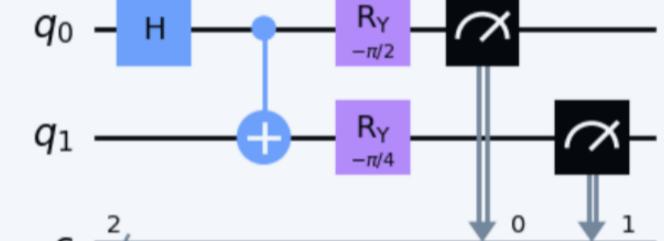
$$rac{1}{\sqrt{2}}(\ket{01}+\ket{10})$$

#### Experiment



## translate





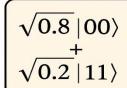
Circuits

#### **Entanglement Level**

Separable

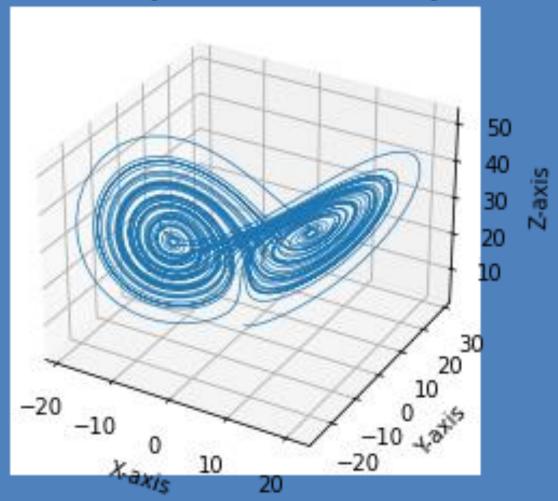
Bell

 $|00\rangle$ 



 $\sqrt{0.6} |00\rangle$  $\sqrt{0.4} \ket{11}$   $\frac{1}{2}(|00\rangle$ |11)

#### ButterFly Effect- Chaos Theory

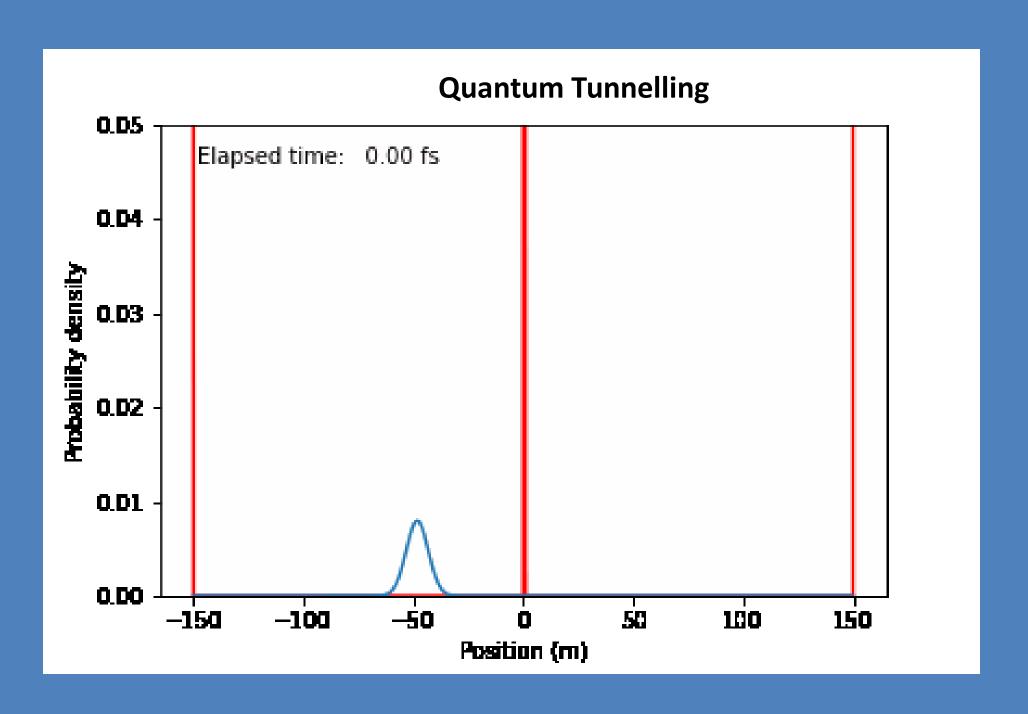


#### Github:

- spacetime-shushmi/Quantum-tunelling
- spacetime-shushmi/Chaos-Theory



Mentor: **Léo Bourdet** 







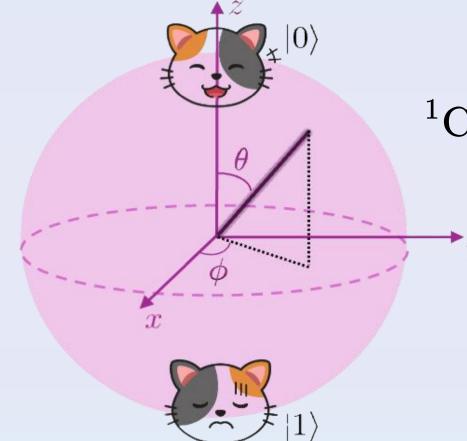
# Can higher order weak values resolve particle presence?

Shushmi Chowdhury<sup>1</sup> and Jörg Götte<sup>2</sup>



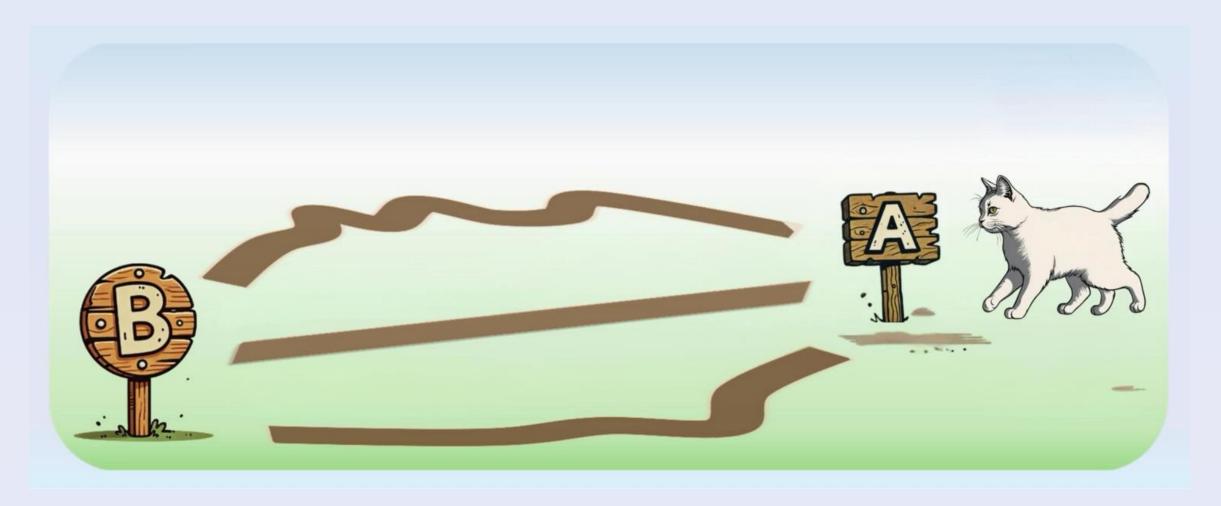
<sup>2</sup>Quantum Theory Group, School of Physics and Astronomy,
University of Glasgow





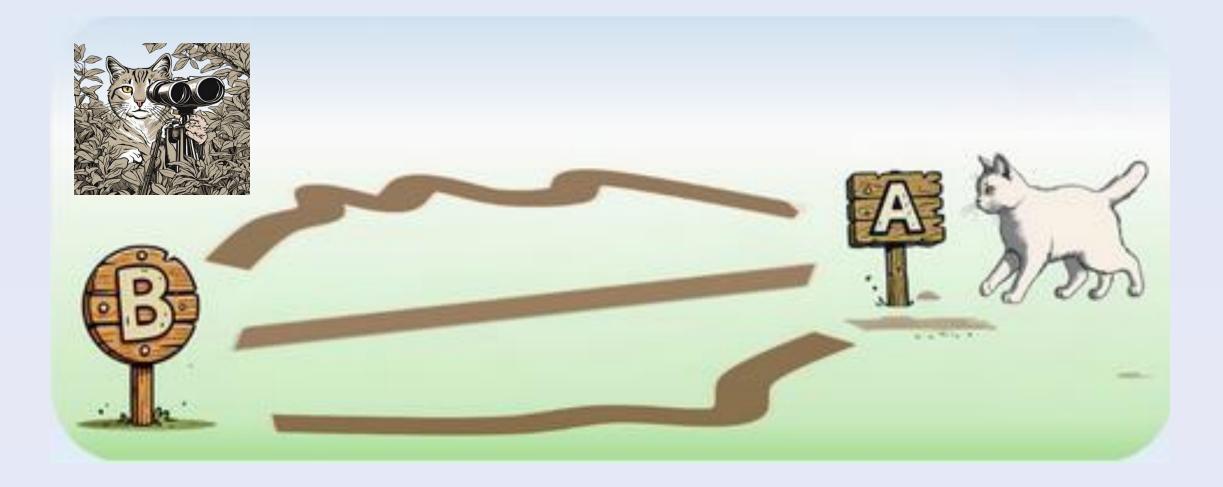
# Motivation

- Quantum particles exhibit both wave-like and particle-like properties, known as wave-particle duality
- When we attempt to measure a quantum particle, its wave-like behavior collapses, and it appears as a particle in a single state, losing its superposition.



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- Under weak measurement, particles can follow discontinuous, unpredictable paths
- By using weak values derived from Feynman's path integrals, we intend to address these unusual paths

# Feynman Path Integrals

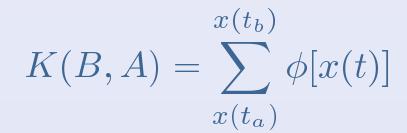
- System traverses all possible paths simultaneously
- Each path has a some phase factor associated with it:

$$\phi[x(t)] = G \exp\left\{\frac{i}{\hbar}S[x(t)]\right\}$$

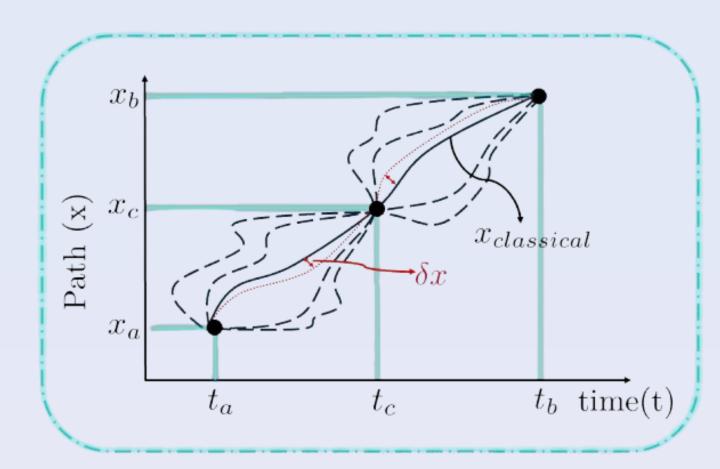
- Phase  $\propto$  S where  $S = \int_{t_a}^{t_b} L(\dot{x}, x, t) dt$
- Lagrangian of particle with mass m in contact with potential V,

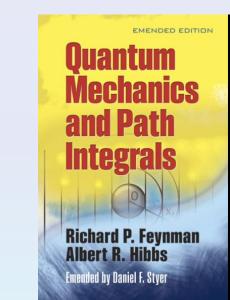
$$L = \frac{m\dot{x}^2}{2} - V(x, t)$$

• Amplitude to go from an initial point, a, to a final point, b:



• Probability to go from  $x(t_A)$  to  $x(t_B)$  is square of total amplitude:  $P(B \leftarrow A) = |K(B, A)|^2$ 





# Time Symmetric Formulation of Quantum Mechanics

- Time-symmetric framework Phys.Rev.134, B1410 Consider Initial and Final states
- Coherence-destroying measurements  $\{A_1, A_2 \dots A_{N-1}A_N\}$





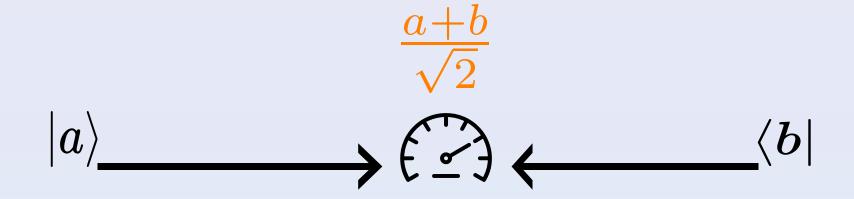






• If order of events is changed between pre- and post-selection, probability remains unchanged:  $P(a_2,\ldots,a_{N-1}|a_1,a_N) = P(\ldots,a_{N-1},\ldots a_2,\ldots |a_1,a_N)$ 

#### Two State Vector Formalism



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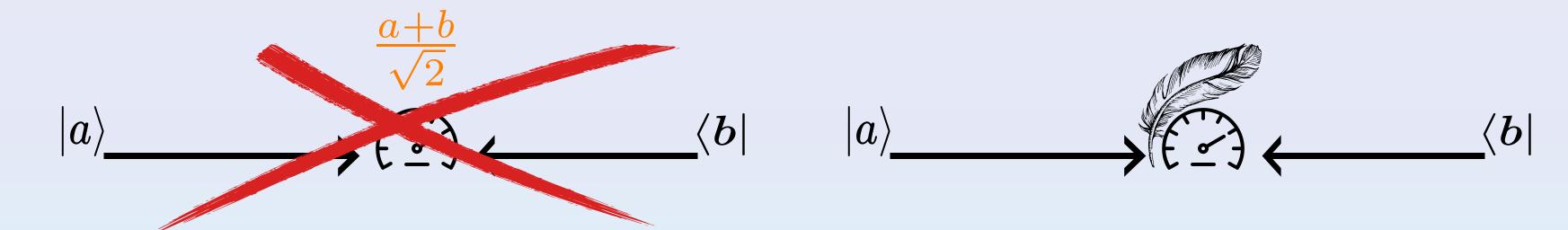




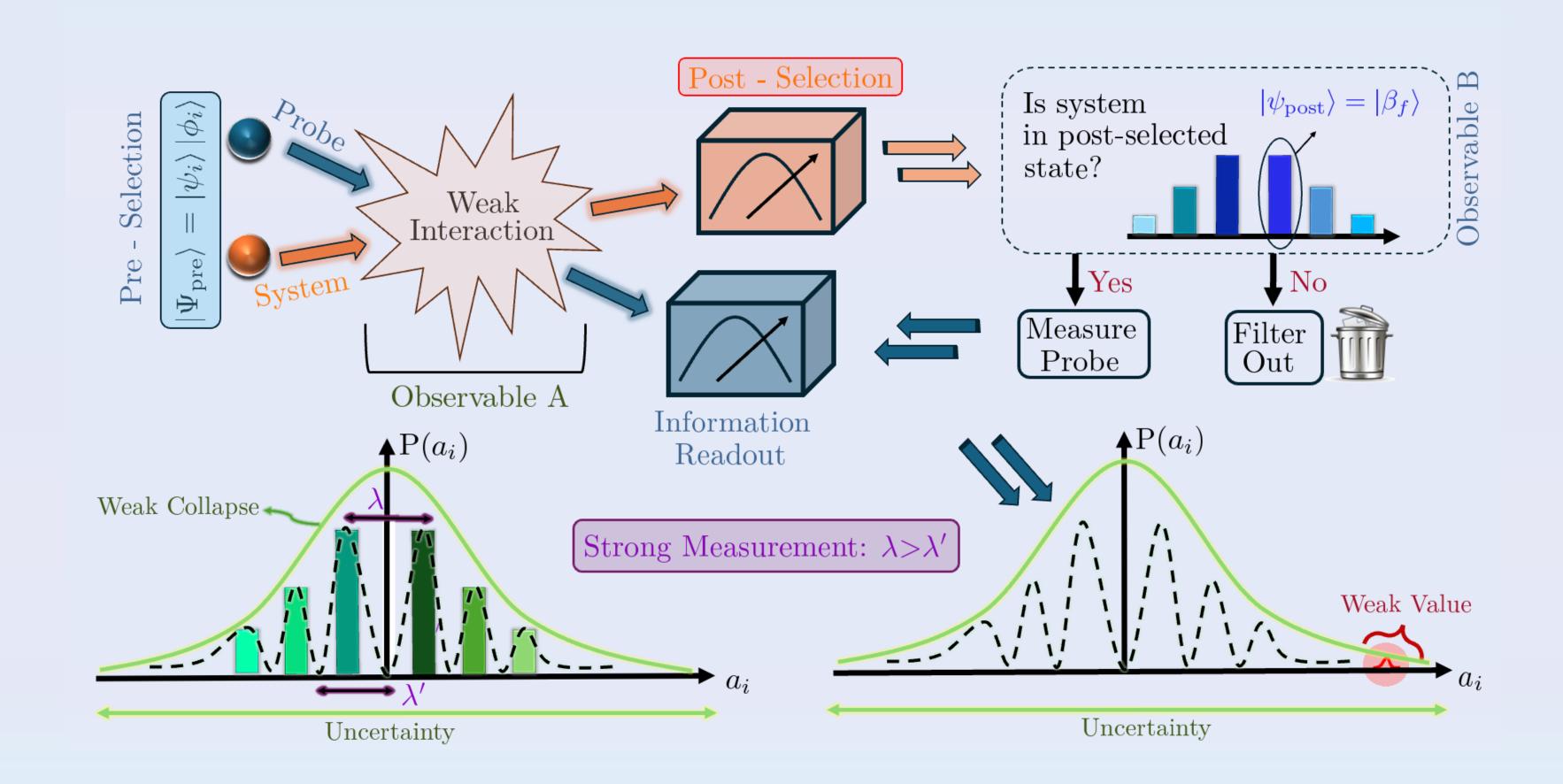
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#### Two State Vector Formalism



## Outline of Weak Measurement Process



## Weak Value

- Average of many slightly perturbed states, with system being in pre- and post-selected states
- Expressed as:  $A^w = \frac{\langle \phi | \hat{A} | \psi \rangle}{\langle \phi | \psi \rangle}$
- $A^w$  can lie outside the range of the allowed eigenvalues of a measurement operator
- Can take complex values

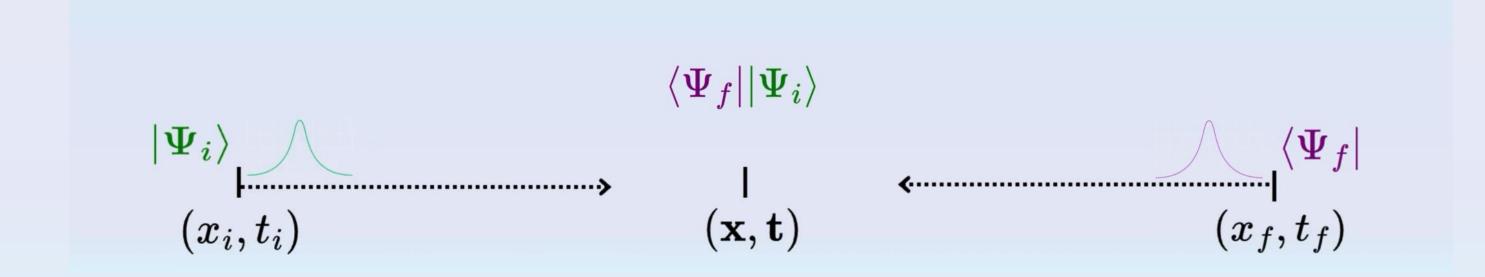
# Deriving Weak Values from Path Integrals

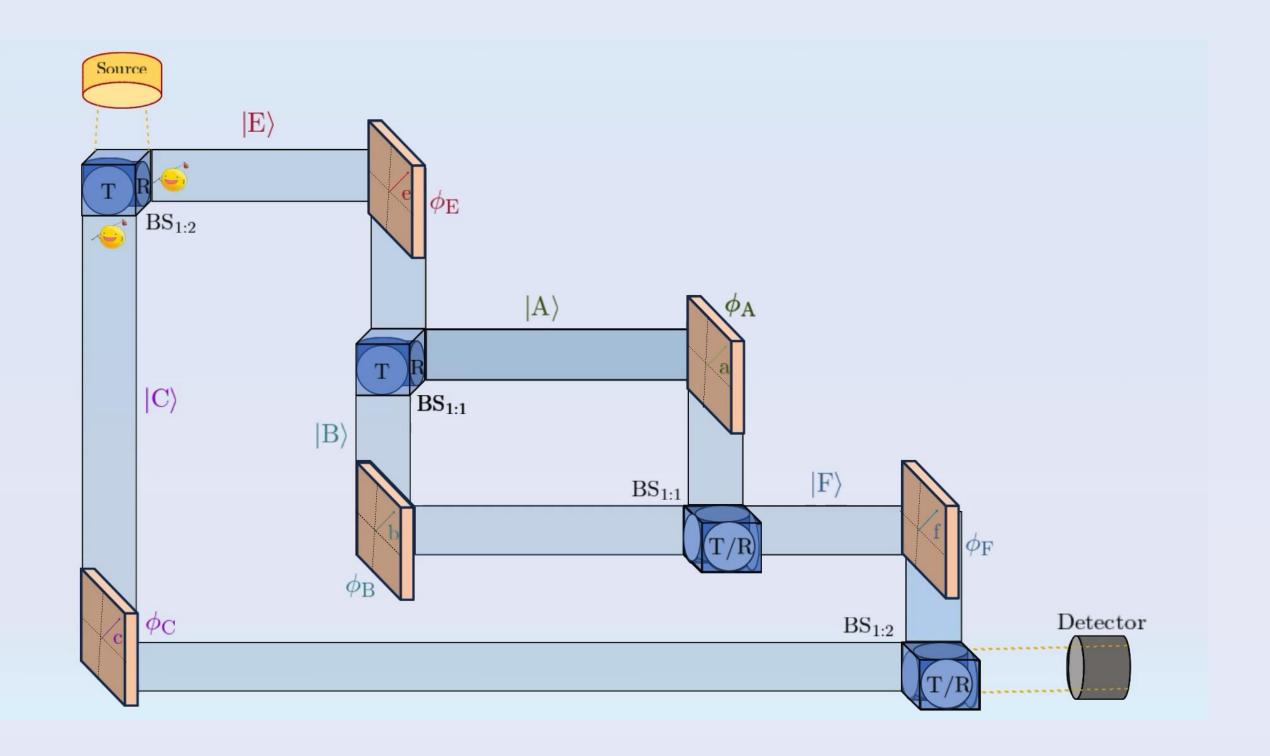
**WEAK VALUES** 

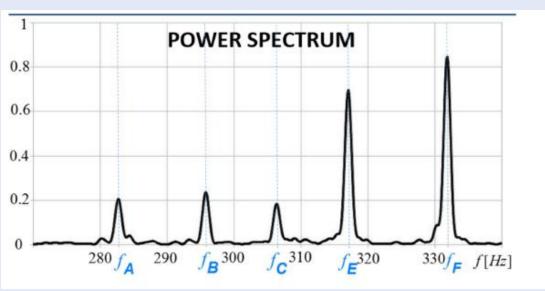


Path Integrals

$$\mathbf{A}^{\mathbf{w}} = \frac{\int dx_i d\mathbf{x} dx_f K_s \Big( x_f, t_f; \mathbf{x}, \mathbf{t} \Big) A(x) \delta(x, \mathbf{X}) K_s \Big( \mathbf{x}, \mathbf{t}; x_i, t_i \Big) \psi(\mathbf{x}, \mathbf{t}) \beta^*(\mathbf{x}, \mathbf{t})}{\int d\mathbf{x} dx_f K_s^* \Big( x_f, t_f; x_i, t_i \Big) \psi(\mathbf{x}, \mathbf{t}) \beta^*(\mathbf{x}, \mathbf{t})}$$



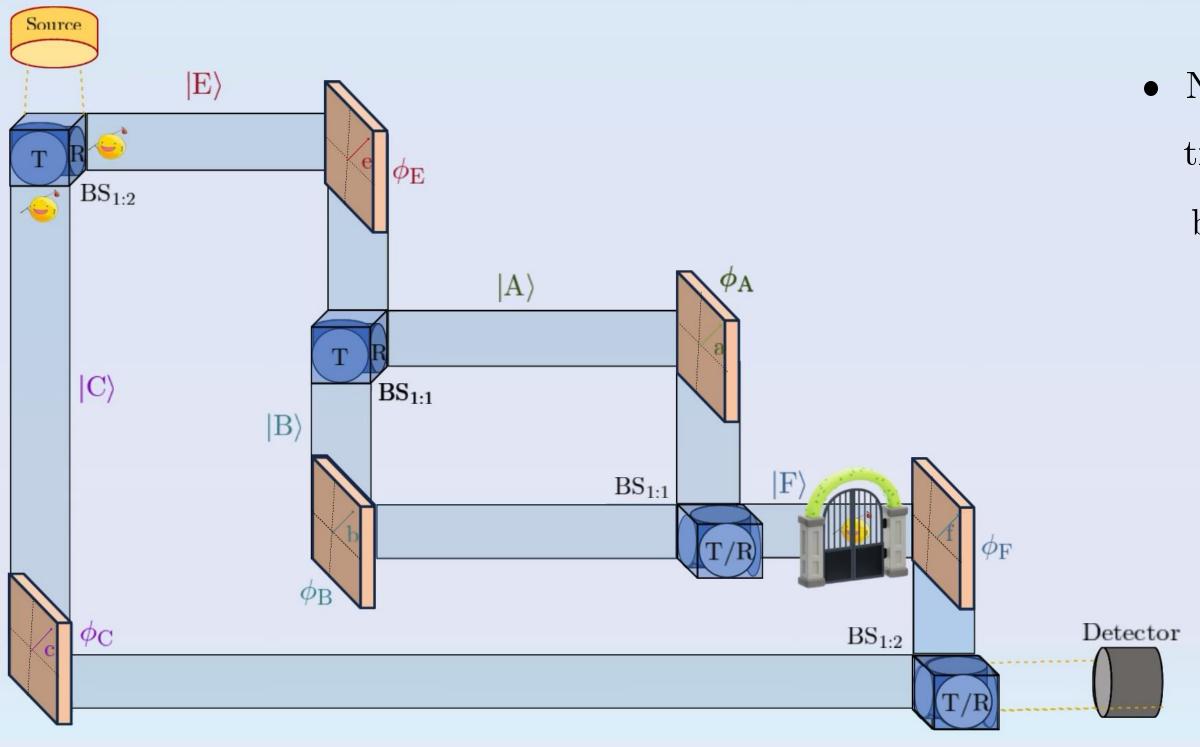




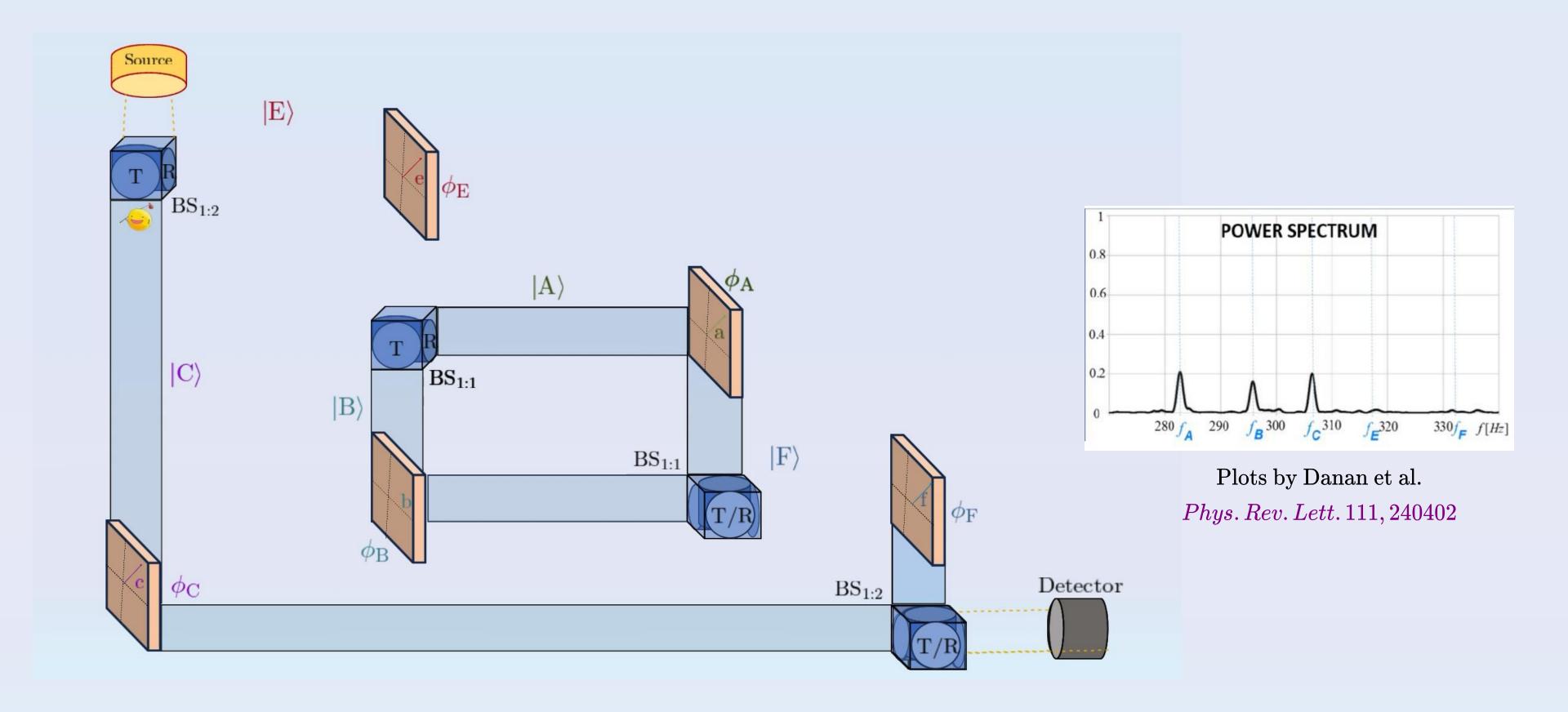
Plots by Danan et al.

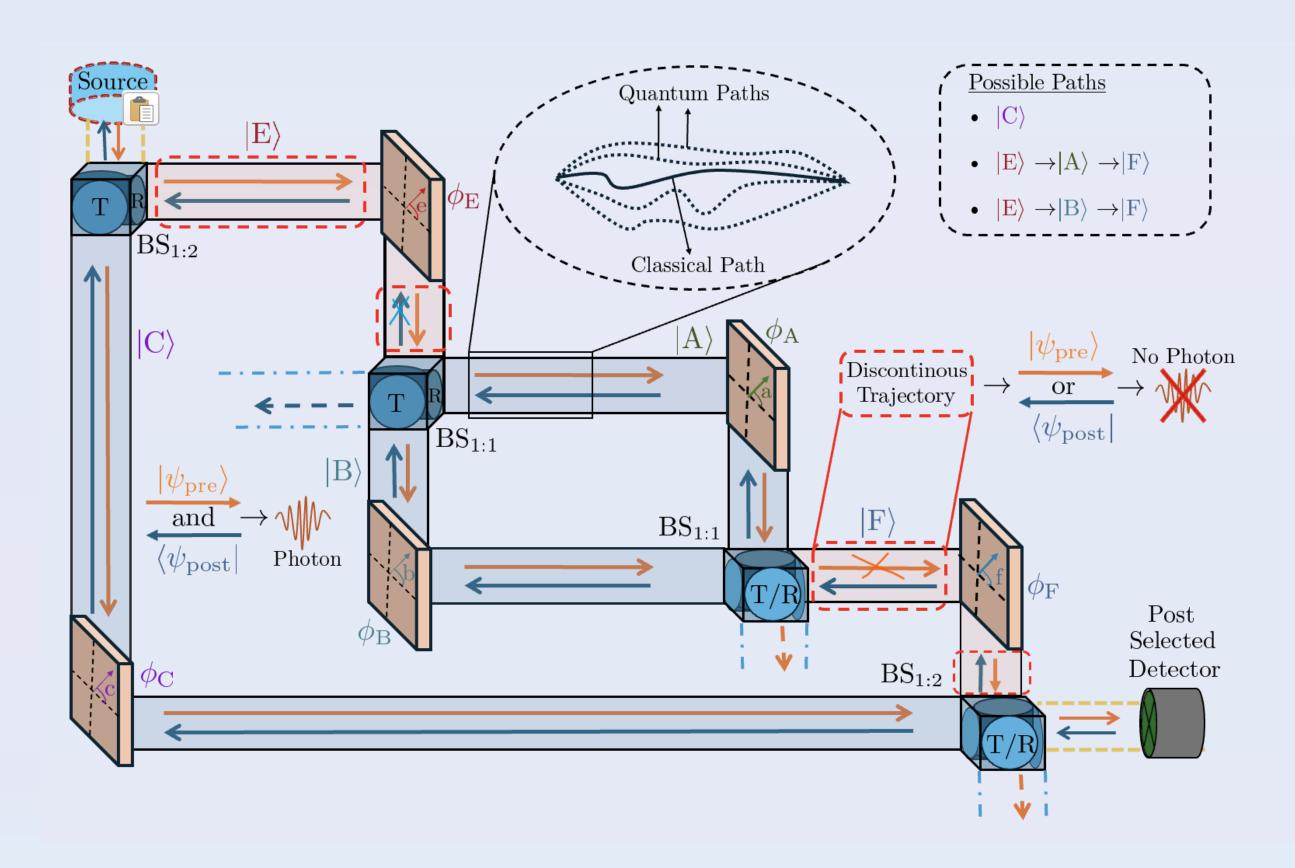
Phys. Rev. Lett. 111, 240402

• Tune mirror B to get destructive interference at second beam splitter in inner interferometer preventing beam from reaching Mirror F



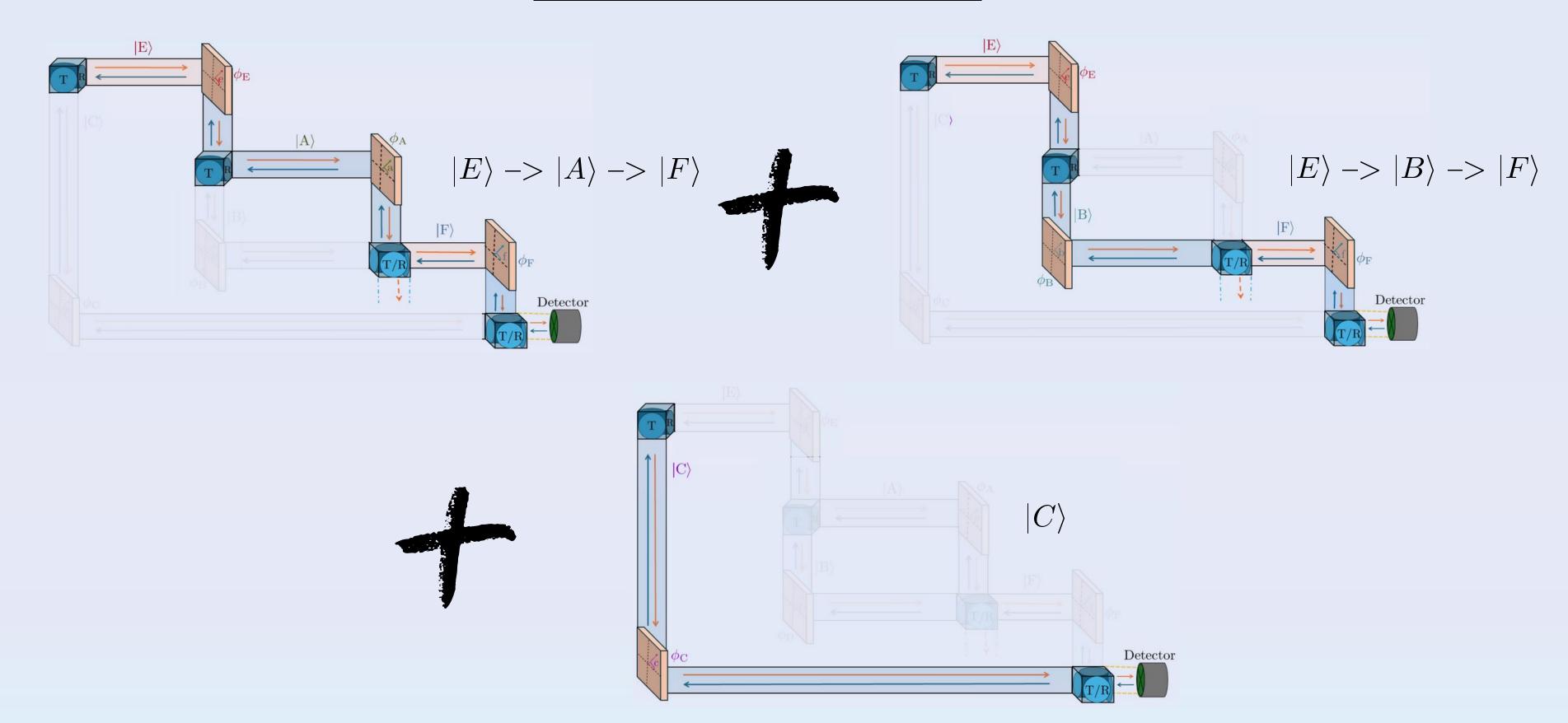
• Natural assumption that photon travelled through  $|C\rangle$  if detected by post-selected detector



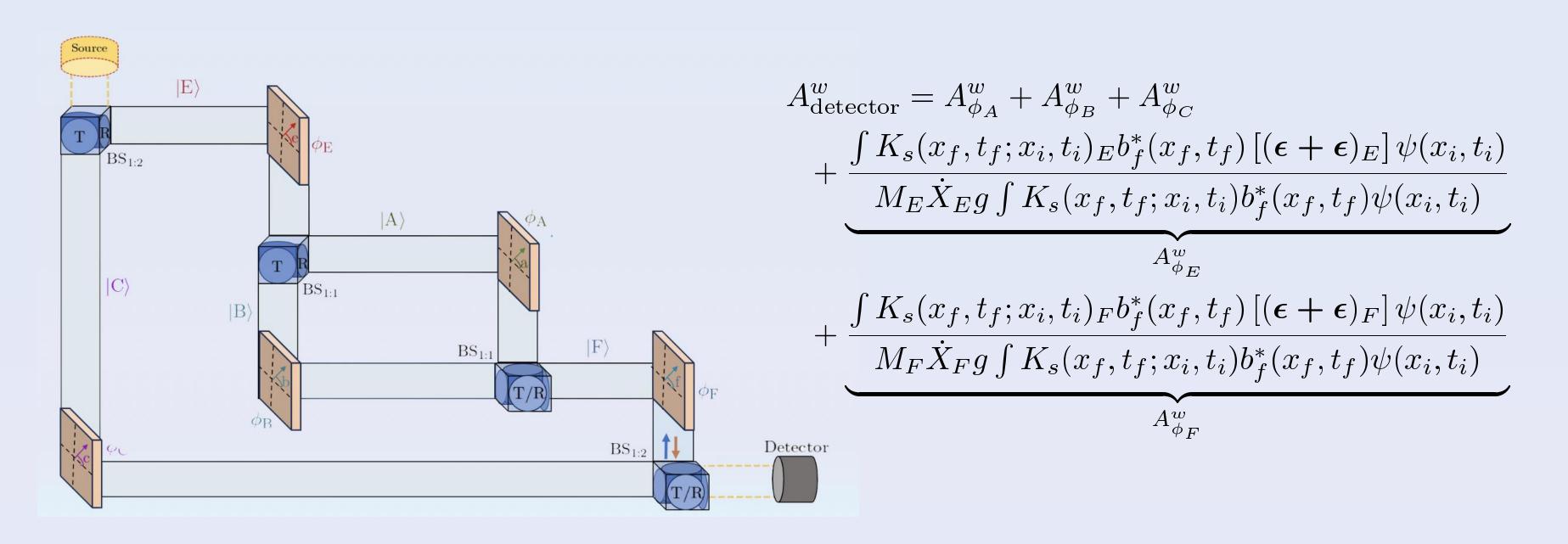


• Two State Vector Formalism

Add all paths (Interference)



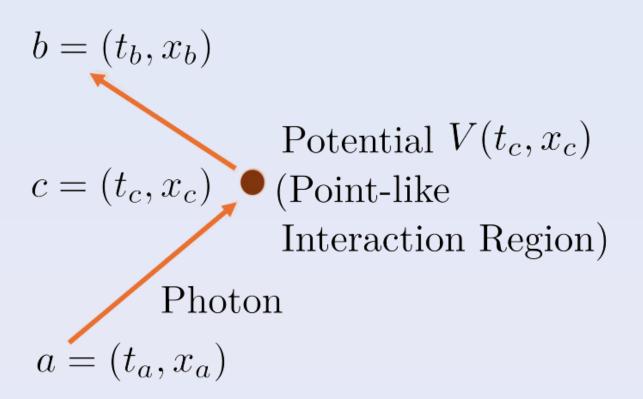
#### Combined Weak Value for Path 1+ Path 2 + Path 3:



If  $\epsilon + (-\epsilon)$  for probes E and F, the weak values disappear

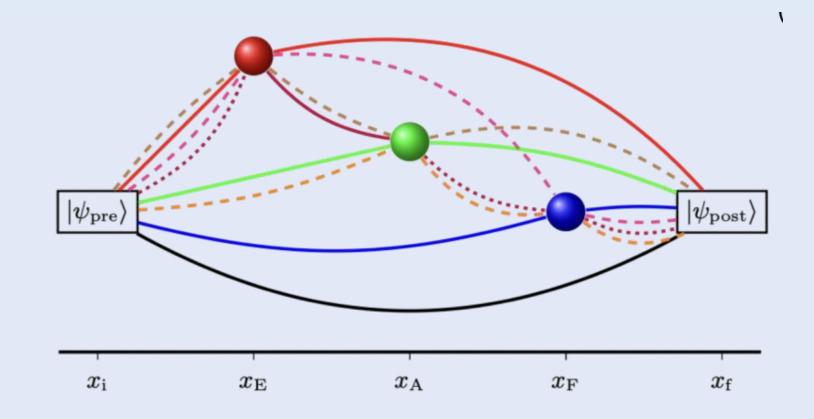
# Higher Order Terms with a Simpler approach

- Interaction treated as particle scattering on encountering a potential
- Paths can be constructed for successive events with amplitudes, k(B, A), multiplying



# Higher Order Terms with a Simpler approach

Path 1' + Path 2' + Path 3' =
$$3 - (\epsilon_A + \epsilon_B + \epsilon_C) - (2\epsilon_E + 2\epsilon_F)$$
1st order
$$+ [(-\epsilon_A - \epsilon_B) (-\epsilon_E - \epsilon_F) + 2 (\epsilon_E \epsilon_F)]$$
2nd order
$$+ (-\epsilon_A - \epsilon_B) \epsilon_E \epsilon_F.$$
3 d order



• excluding second-order terms isn't necessarily justified, so we still have contributions from  $\epsilon_E$  and  $\epsilon_F$  in the second order.

• Non-trivial to associate discontinuous paths with 0 weak values without taking into account the second and higher-order terms

# Higher Order Terms with a Simpler approach

• Second-order terms could be ascribed as secondary presence arXiv:1312.7566 [quant-ph]

#### **Quantum Cheshire Cats**

Yakir Aharonov,<sup>1,2</sup> Sandu Popescu,<sup>3</sup> Daniel Rohrlich,<sup>4</sup> and Paul Skrzypczyk<sup>5</sup>

<sup>1</sup>Tel Aviv University, School of Physics and Astronomy, Tel Aviv 69978, Israel
<sup>2</sup>Schmid College of Science, Chapman University, 1 University Drive, Orange, CA 92866, USA
<sup>3</sup>H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, United Kingdom
<sup>4</sup>Physics Department, Ben Gurion University of the Negev, Beersheba, Israel
<sup>5</sup>Department of Applied Mathematics and Theoretical Physics, University of Cambridge,
Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, United Kingdom

In this paper we present a quantum Cheshire Cat. In a pre- and post-selected experiment we find the Cat in one place, and its grin in another. The Cat is a photon, while the grin is its circular polarization.

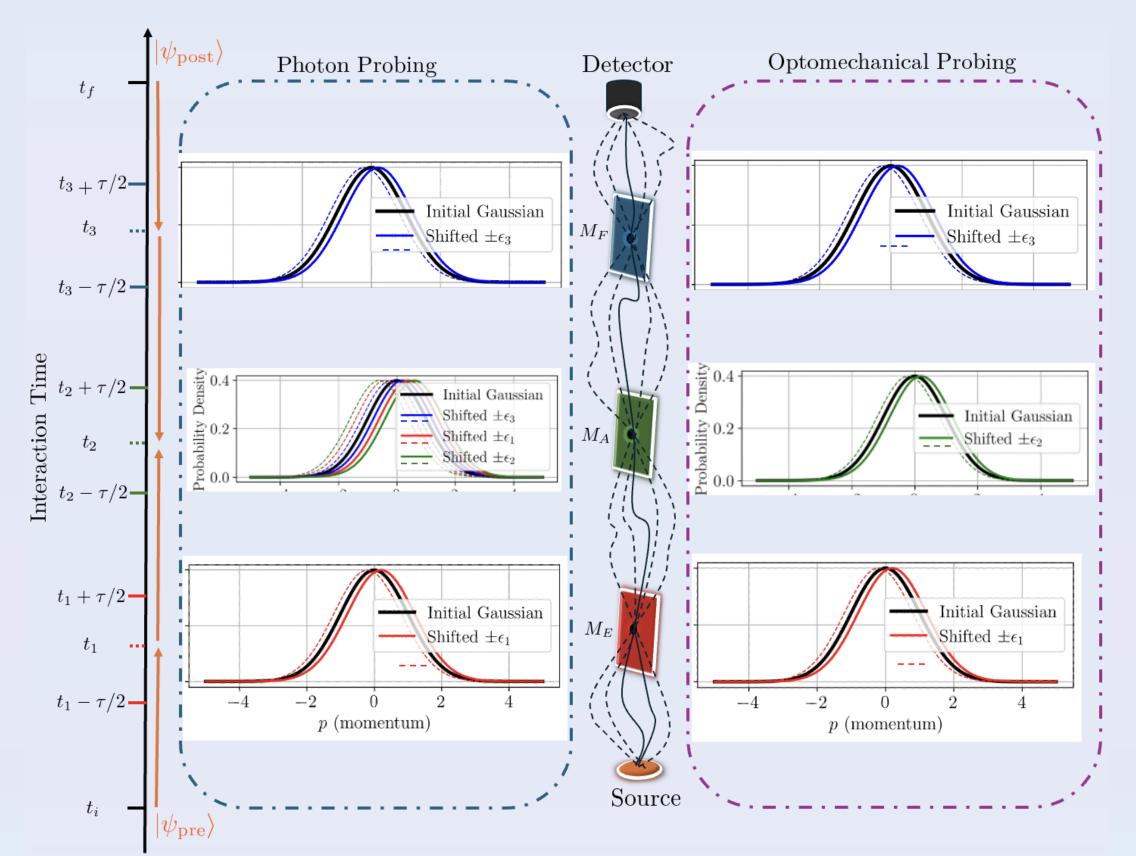
#### I. INTRODUCTION

'All right,' said the Cat; and this time it vanished quite slowly, beginning with the end of the tail, and ending with the grin, which remained some time after the rest of it had gone.

'Well! I've often seen a cat without a grin,' thought Alice, 'but a grin without a cat! It's the most curious thing I ever saw in my life!'

No wonder Alice is surprised. In real life, assuming that cats do indeed grin, the grin is a *property* of the cat – it makes no sense to think of a grin without a cat. And this goes for almost all physical properties. Polarization is a property of photons; it makes no sense to have polarization without a photon. Yet, as we will show here, in the curious way of quantum mechanics, photor polarization may exist where there is no photon at all. At least this is the story that quantum mechanics tells via measurements on a pre- and post-selected ensemble.

#### Experimental Proposal - Optomechanical Treatment



- May reduce discrepancies between theory and experiment
- Mirrors mounted on harmonic mechanical oscillators, initially on ground state
- Oscillator move from its ground state given enough photons interact
- Feedback post-selection

# Conclusion

#### Higher order weak values for paths in nested Mach-Zender interferometers

Shushmi Chowdhury, Jörg B. Götte

We consider weak values in the Feynman propagator framework, to gain a broader understanding of their interpretation in terms of path integrals. In particular, we examine the phenomenon of seemingly discontinuous paths that particles take in nested Mach-Zender interferometer experiments. We extend on existing path integral approaches for weak values by deriving expressions to model a sequence of weak measurements, and study the probe shifts across the different branches of a weak value interferometer. We apply this to scrutinise two scenarios of interest, one which treats photons as measurement apparatus via their spatial projection operators, and the second treating mirrors as probes.

Comments: 14 pages, 7 figures plus one in appendix
Subjects: Quantum Physics (quant-ph)
Cite as: arXiv:2407.06989 [quant-ph]

(or arXiv:2407.06989v1 [quant-ph] for this version)

https://doi.org/10.48550/arXiv.2407.06989

#### Submission history

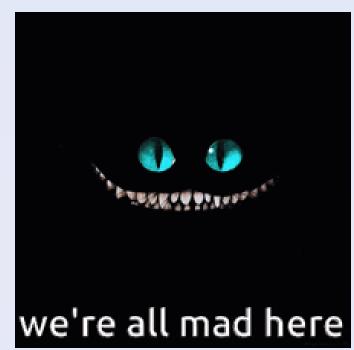
From: Jörg Götte [view email]

[v1] Tue, 9 Jul 2024 16:04:05 UTC (4,036 KB)



Just the cool ideas, not pages of derivation.

Because why not -

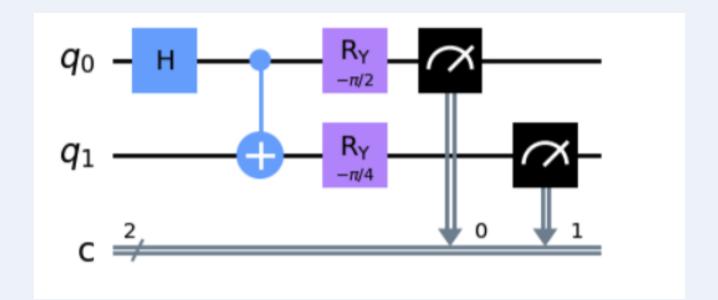


Time for some Quantum Computing...

# We have heard a lot about digital Quantum Computers but what about Analog?

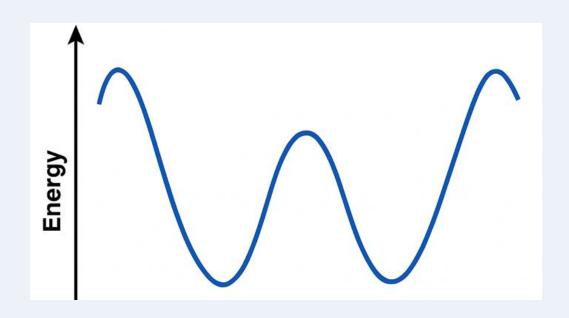


 Like programming a classical computer, but with qubits.





System naturally evolves toward the lowest-energy state.





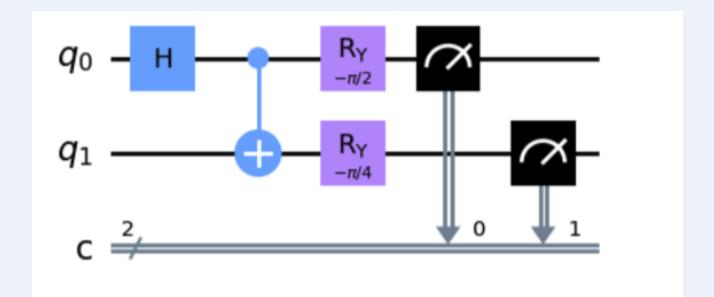




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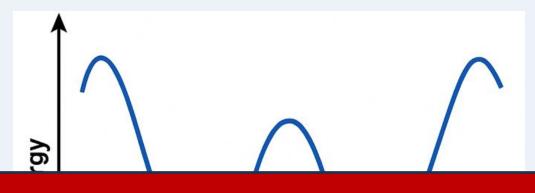


 Like programming a classical computer, but with qubits.





System naturally evolves toward the lowest-energy state.



- Not universal
- Only specific problems can be encoded
- Can run quantum algorithms like Shors, Grovers, QAOA

Computing Inc.

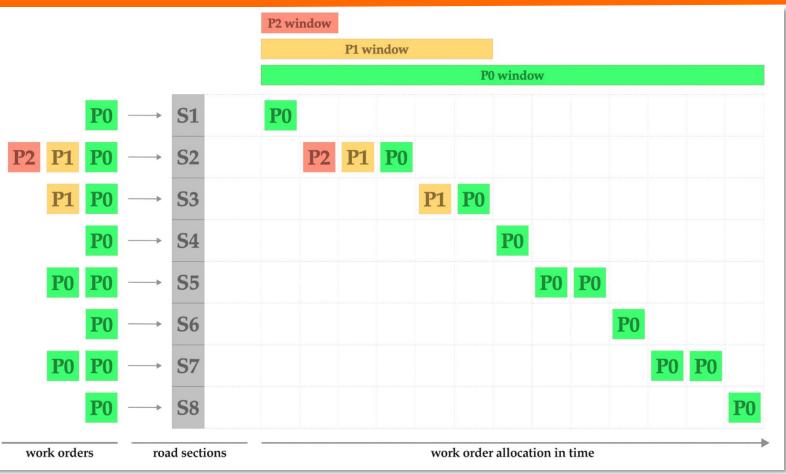




# UK Quantum Hackathon

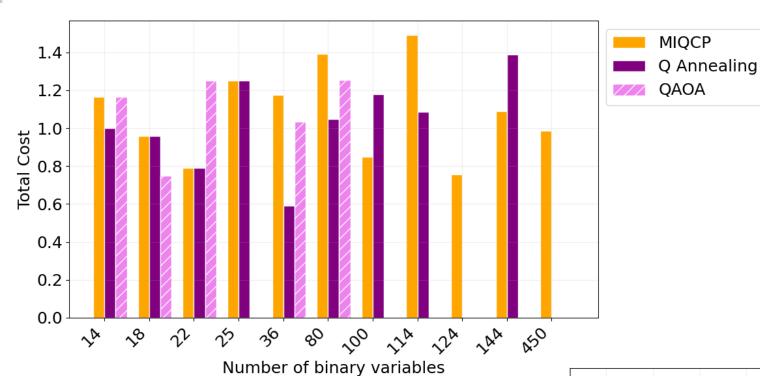
**QUBO** 







- More vehicles -> more road wear
- Tight budgets -> need smarter planning
- Scheduling road maintenance is a complex
- combinatorial problem, hard to find optimal
- solutions at increasing scales
- How can quantum hardware help with this issue?









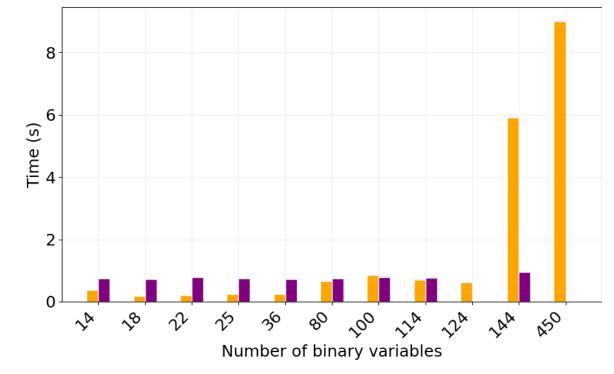


Iteratively solving QUBO using D-Wave

5000 runs, 30 micro-seconds

| Quantum<br>Hardware | Qiskit<br>IBM | CZephyr) |
|---------------------|---------------|----------|
| Qubits              | 156           | 4596     |





UK Quantum Hackathon \_





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MIQCP Q Annealing QAOA





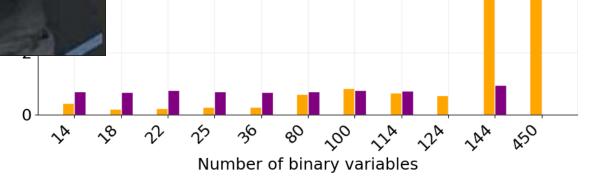


QUBO (

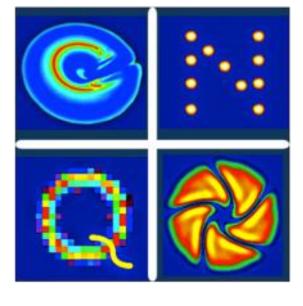
| Quantum<br>Hardware | Qiskit IBM |
|---------------------|------------|
| Qubits              | 156        |









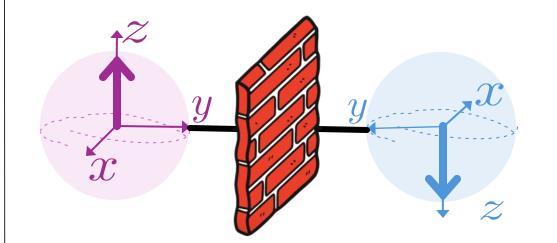


# Benchmarking coherence and noise through domain-wall distributions in quantum annealers

Shushmi Chowdhury<sup>1</sup>, Asa Hopkins<sup>1</sup>, Nicholas Chancellor<sup>2</sup> and Viv Kendon <sup>1</sup>

<sup>1</sup>CNQO, Department of Physics, University of Strathclyde

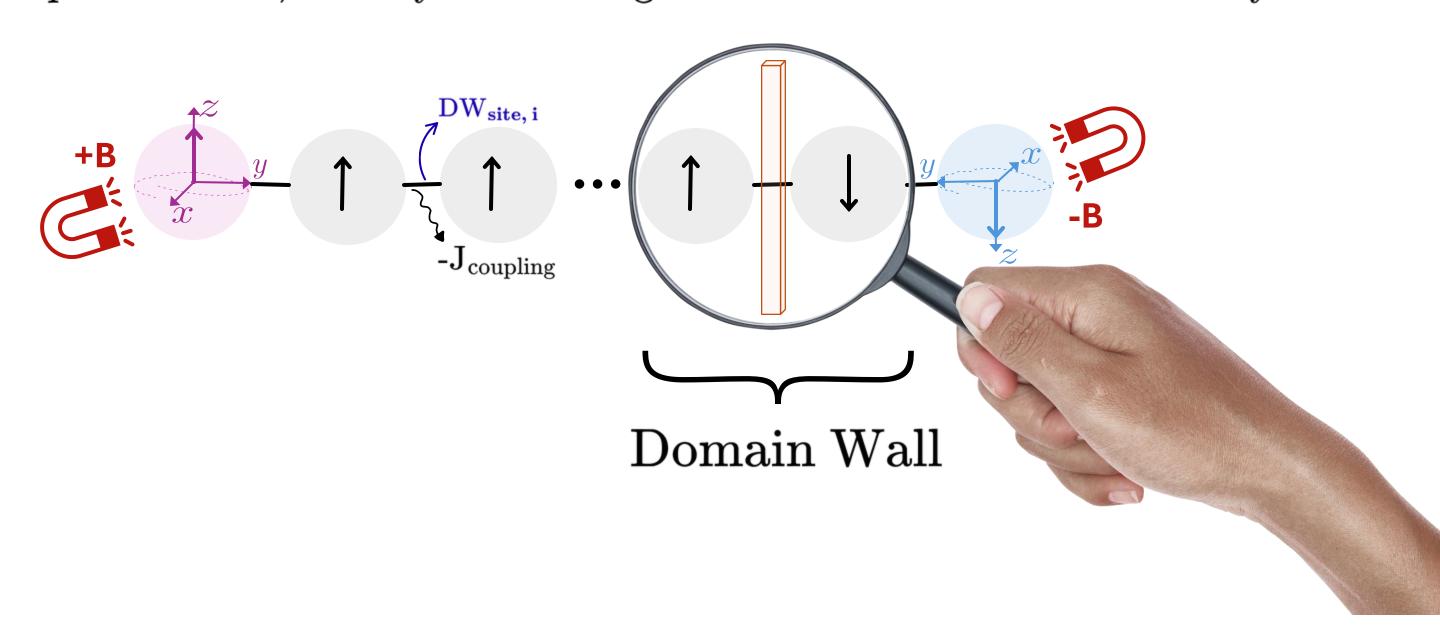
<sup>2</sup>School of Computing, Newcastle University





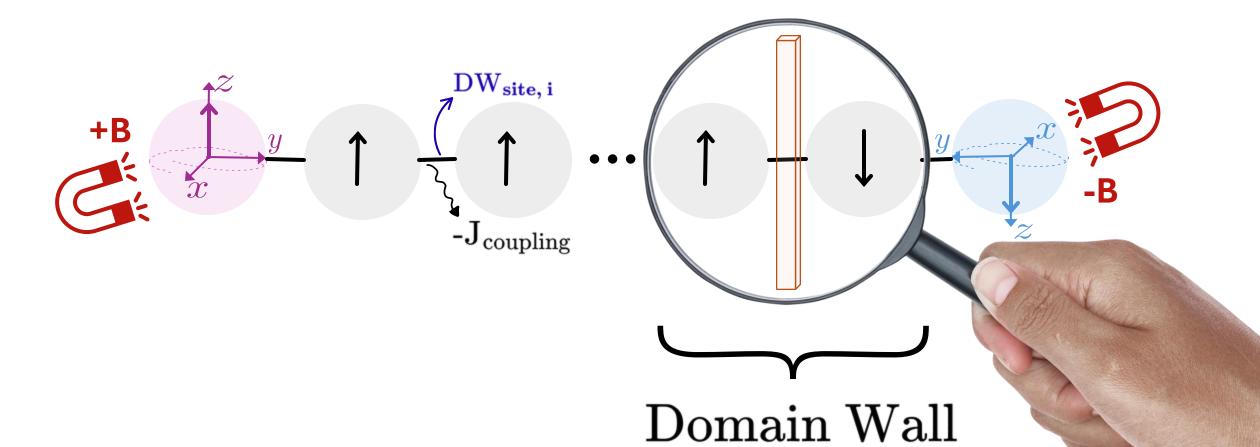
## Motivation

A finite ferromagnetic spin chain in which the spins at the two ends of the chain are constrained to have opposite polarisations, thereby introducing an inherent frustration into the system.



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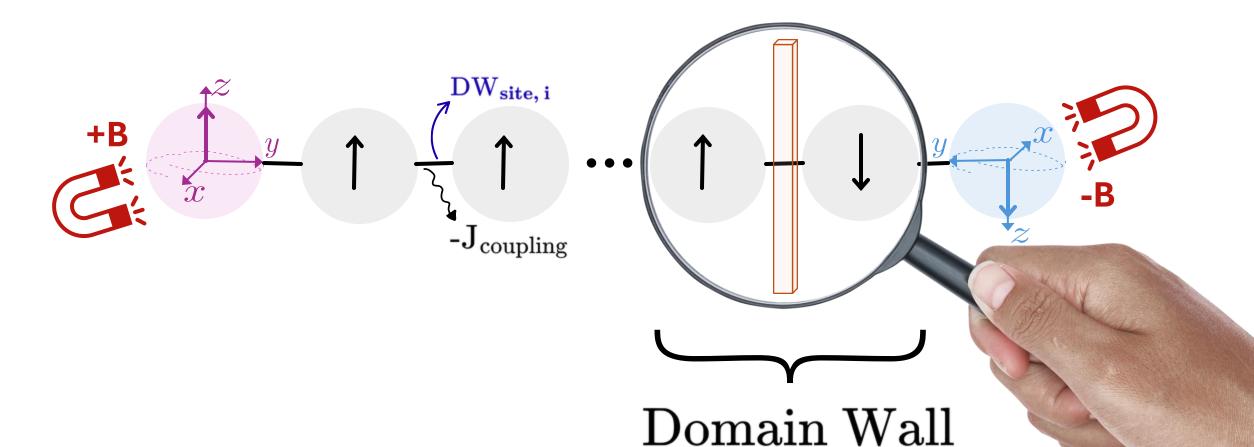
A finite ferromagnetic spin chain in which the spins at the two ends of the chain are constrained to have opposite polarisations, thereby introducing an inherent frustration into the system.



Problem formulated using:  $H_{Ising} = J \sum_{i=1}^{N} -\sigma_i^z \sigma_{i+1} + h(\sigma_i^z \_\sigma_{N+1}^z)$ 

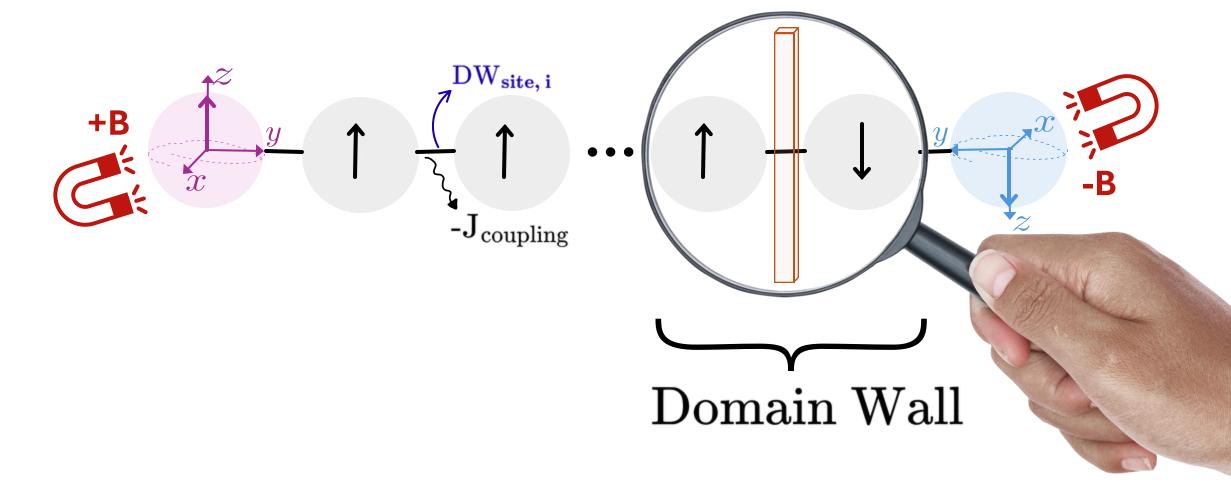
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## Motivation



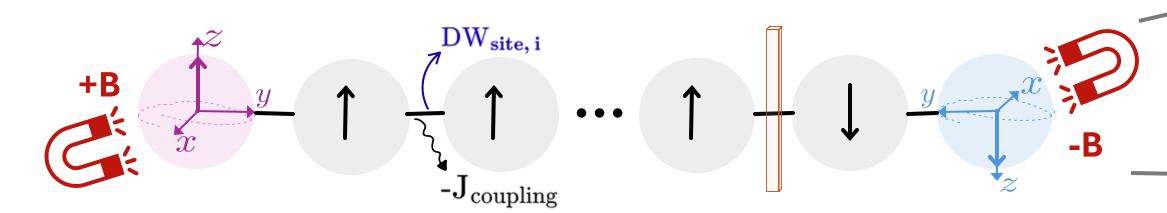


Why we care: The domain wall location reflects how the system resolves frustration

What it tells us: • Domain-wall distributions provide a simple probe of coherence and noise.

- Short Ising chains offer a minimal model for benchmarking device quality.
- Shape of the domain-wall distribution indicates regime of the device, classical to coherent.

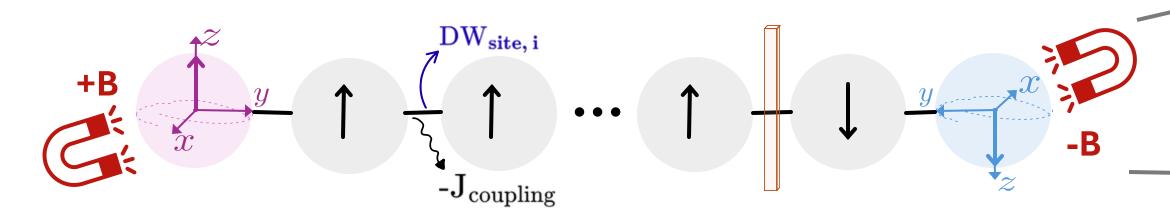
## The Set-up



D-Wave implements transverse field Ising Model (TFIM):

$$egin{aligned} H(t) &= -A(t) \sum_{i=1}^{n} \sigma_i^x + B(t) rac{H_{Ising}}{I_{sing}} \ &= J \sum_{i=1}^{N} -\sigma_i^z \sigma_{i+1} + h(\sigma_i^z \sigma_{N+1}^z) \end{aligned}$$

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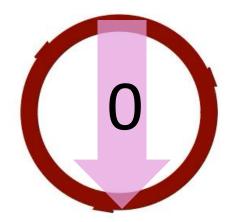
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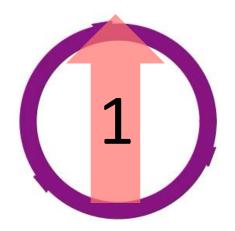
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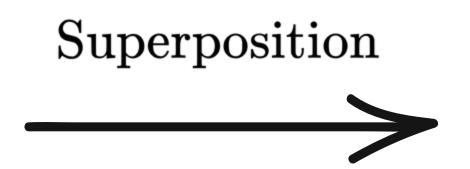


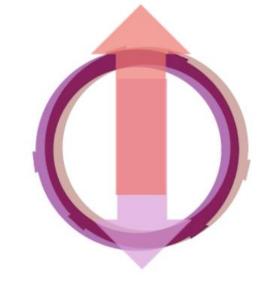


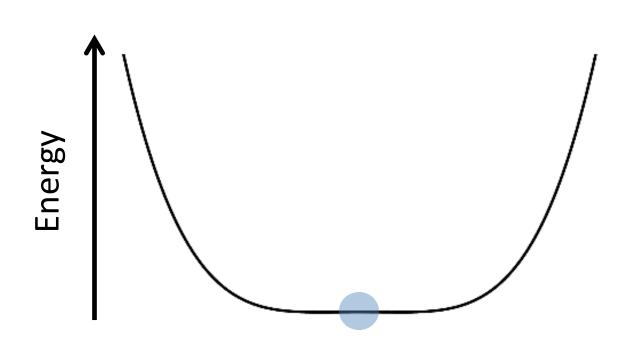
# Quantum Annealing

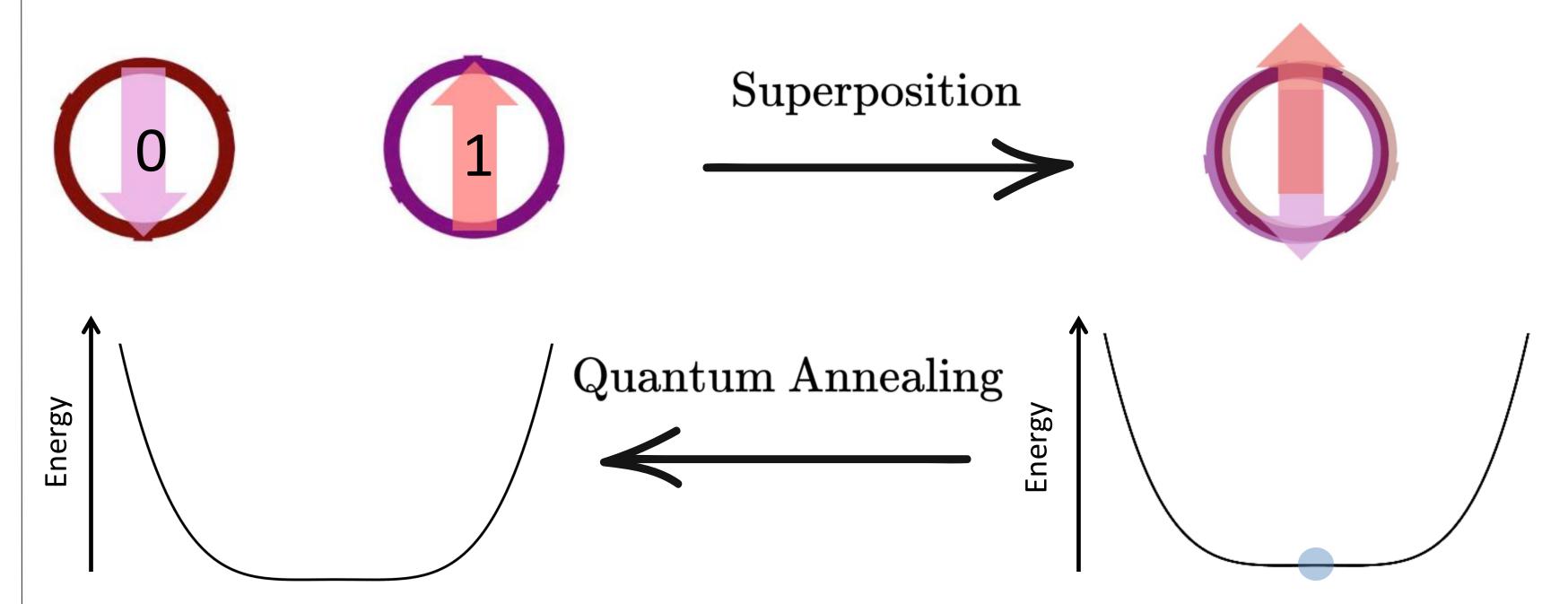


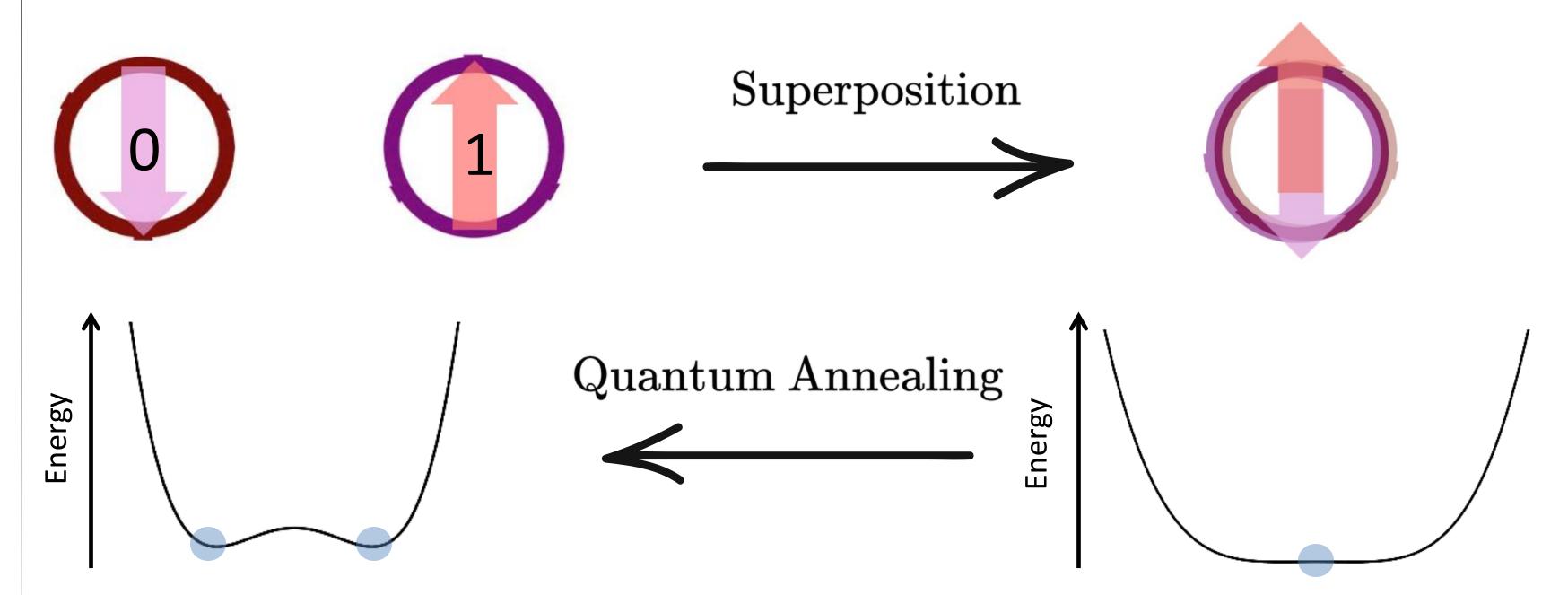


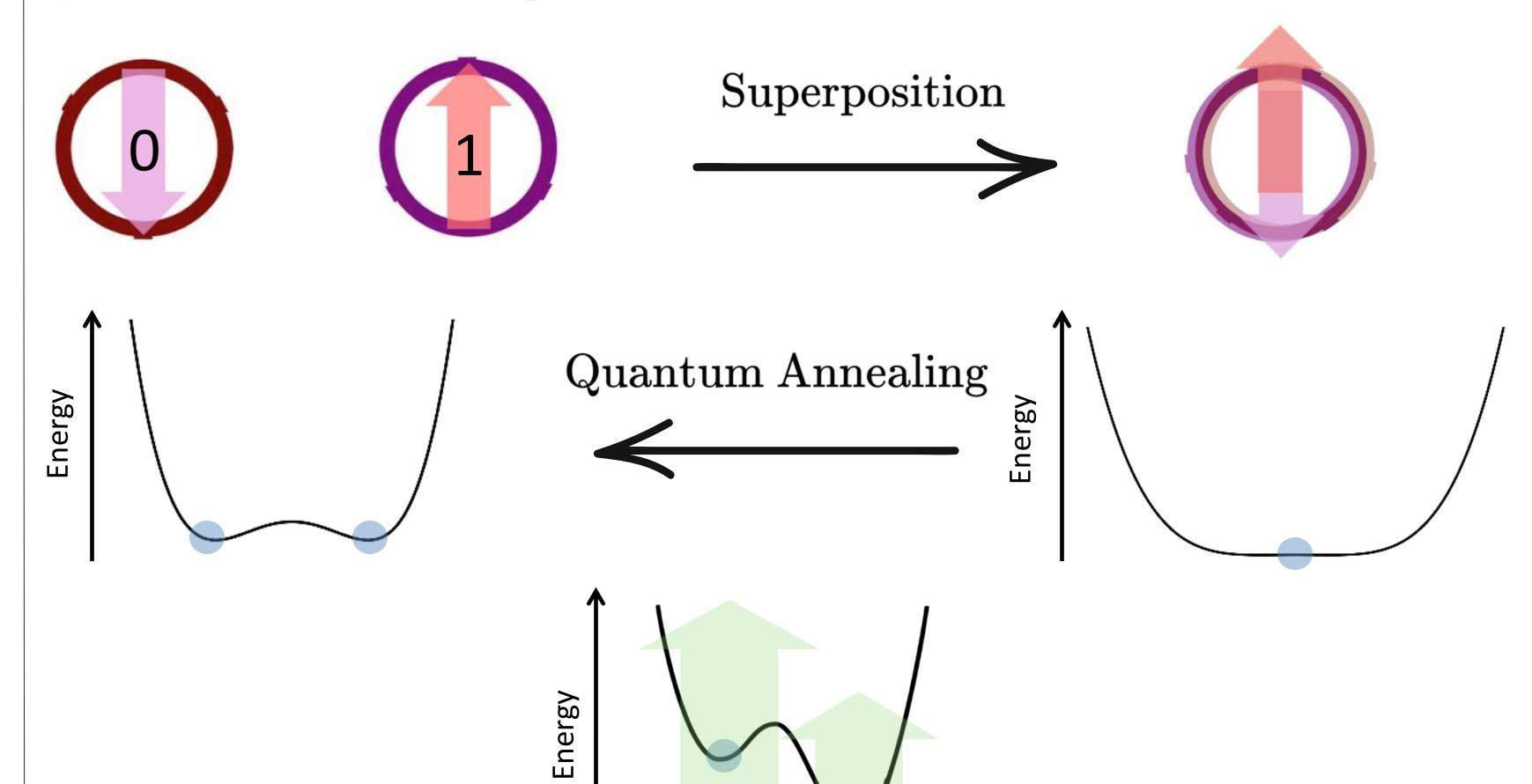


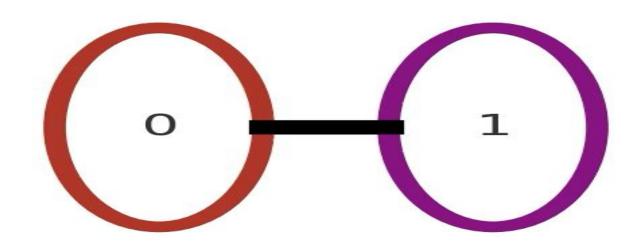


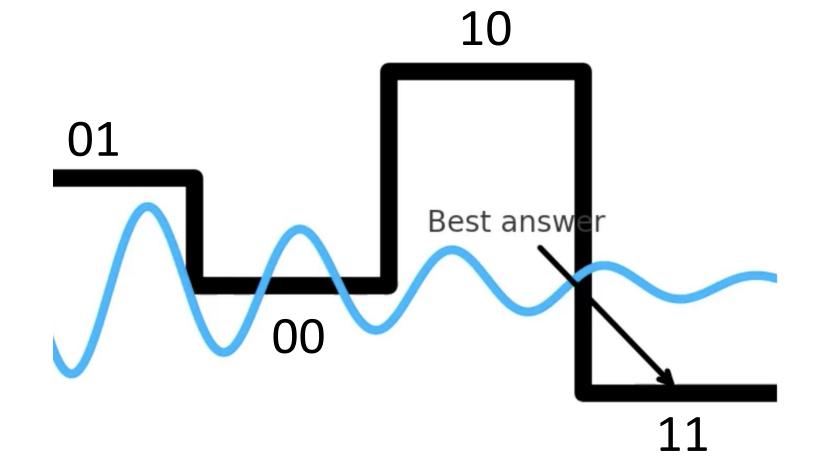




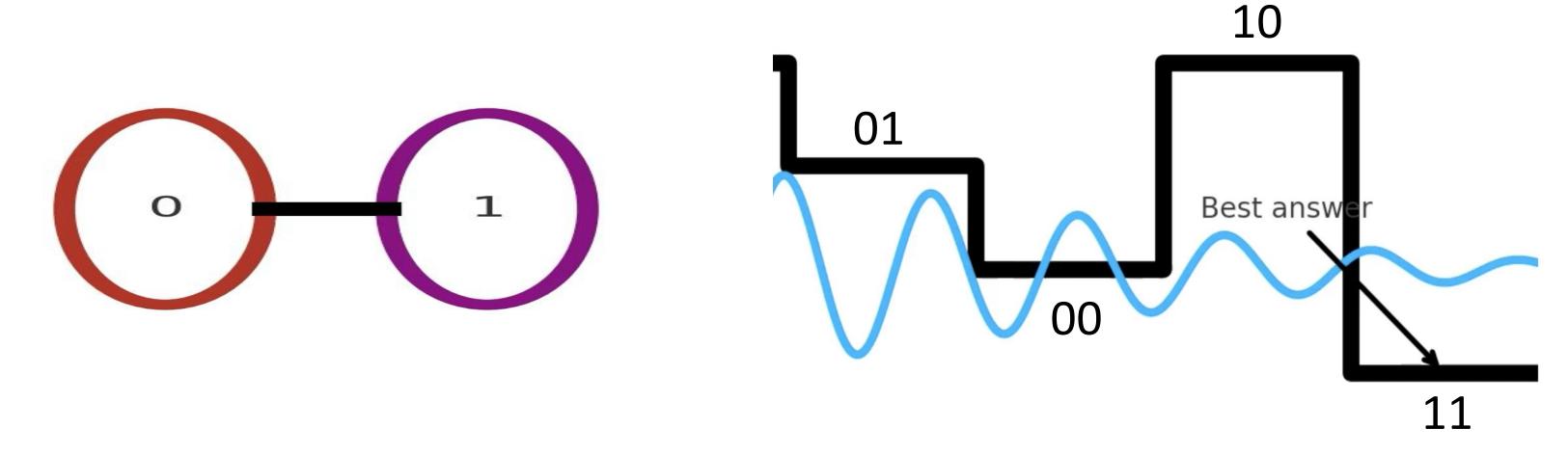








# Quantum Annealing



Many Qubit Coupling



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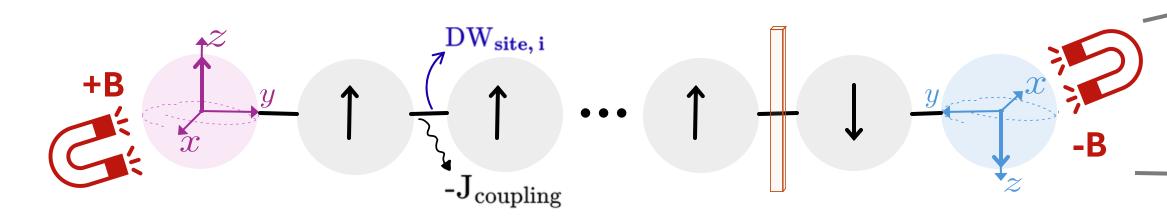
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### Chancellor et. al. npj Quantum Information, 8, 73

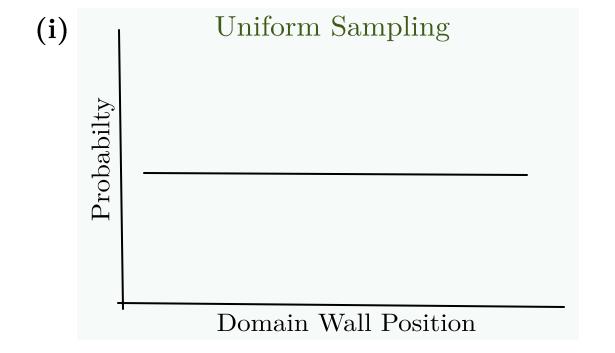
# Previous work



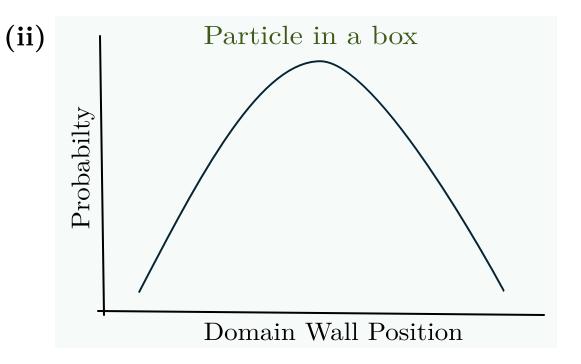
D-Wave implements transverse field Ising Model (TFIM):

$$egin{aligned} H(t) &= -A(t) \sum_{i=1}^{n} \sigma_i^x + B(t) rac{H_{Ising}}{I_{sing}} \ &= J \sum_{i=1}^{N} -\sigma_i^z \sigma_{i+1} + h(\sigma_i^z - \sigma_{N+1}^z) \end{aligned}$$





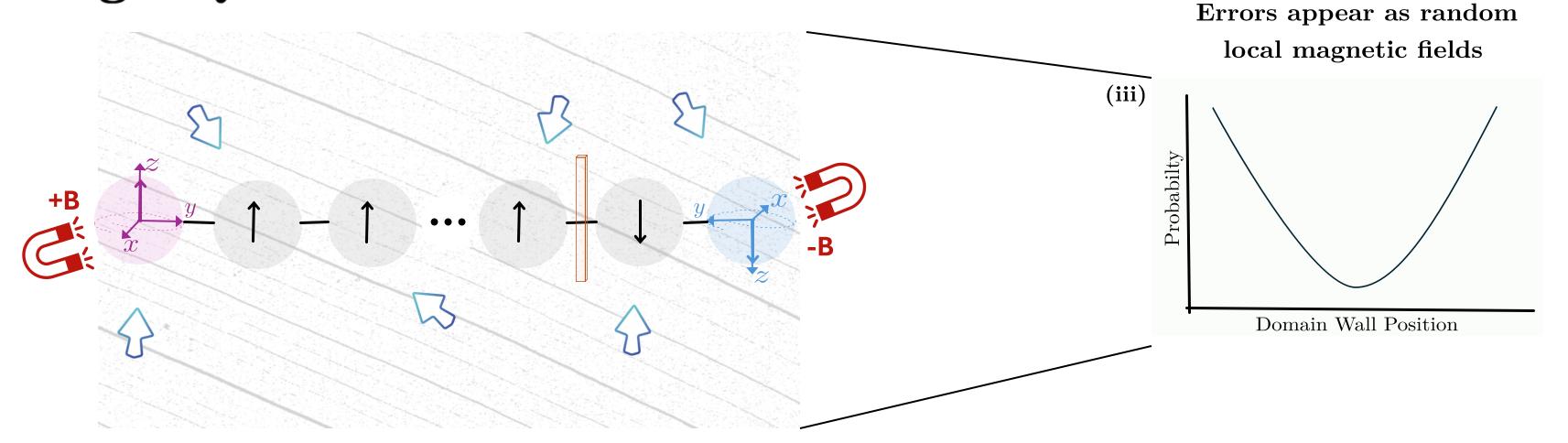
**Expected Result** in absence of noise



# Errors appear as random local magnetic fields Algorithm Algorithm

• Control errors in the single qubit regime arise due to a device implementing a Hamiltonian different from the intended one.

# Single Qubit Control Errors



- Control errors in the single qubit regime arise due to a device implementing a Hamiltonian different from the intended one.
- Add to the Hamiltonian, the following control errors:  $H_{fields} = \sum_i \epsilon_i \sigma_i^z$
- The Boltzmann distribution is taken over the random-field–induced energy shifts within the

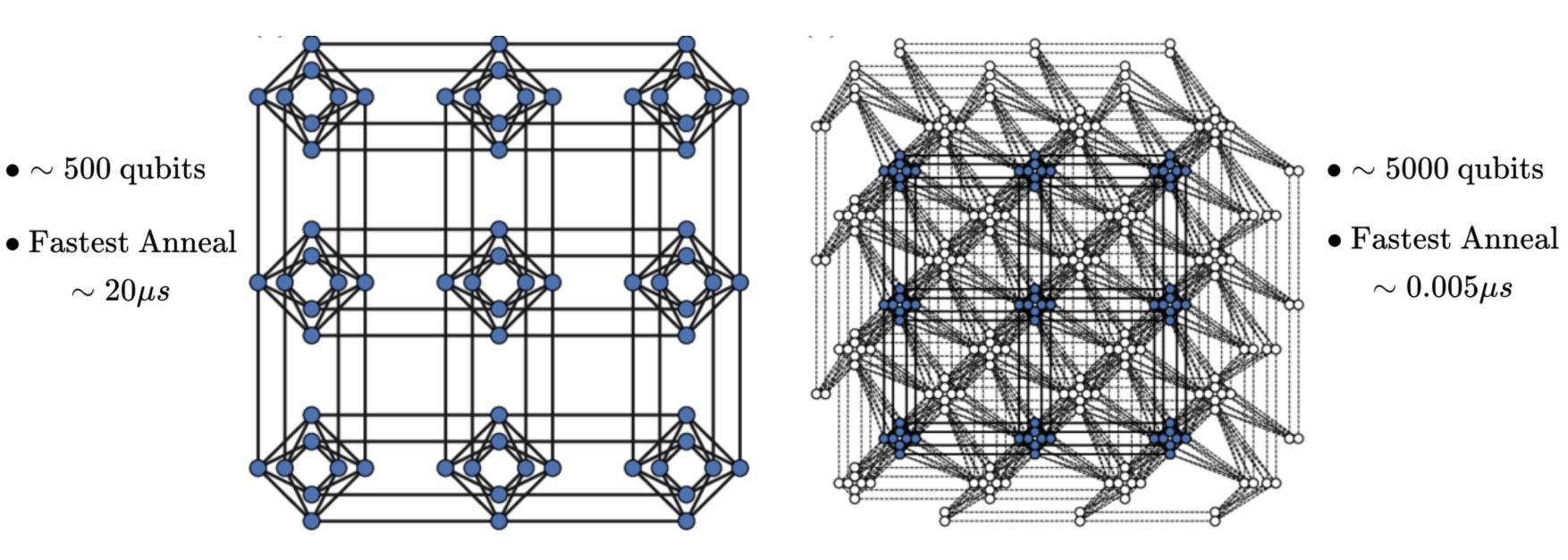
$$ext{degenerate ground-state manifold}: P_n = rac{e^{-eta \sum_{i=1}^n \epsilon_i}}{Z}$$

 $\sim 20 \mu s$ 

 $\times$ 

# What we set out to do...

We study the U-shaped domain wall distribution from an Ising chain with anti-parallel boundaries on the D-Wave Two (Chimera) -npj Quantum Inf. 8, 73 (2022)- and extend the analysis to the newer Advantage 4.1 (Pegasus) QPU.



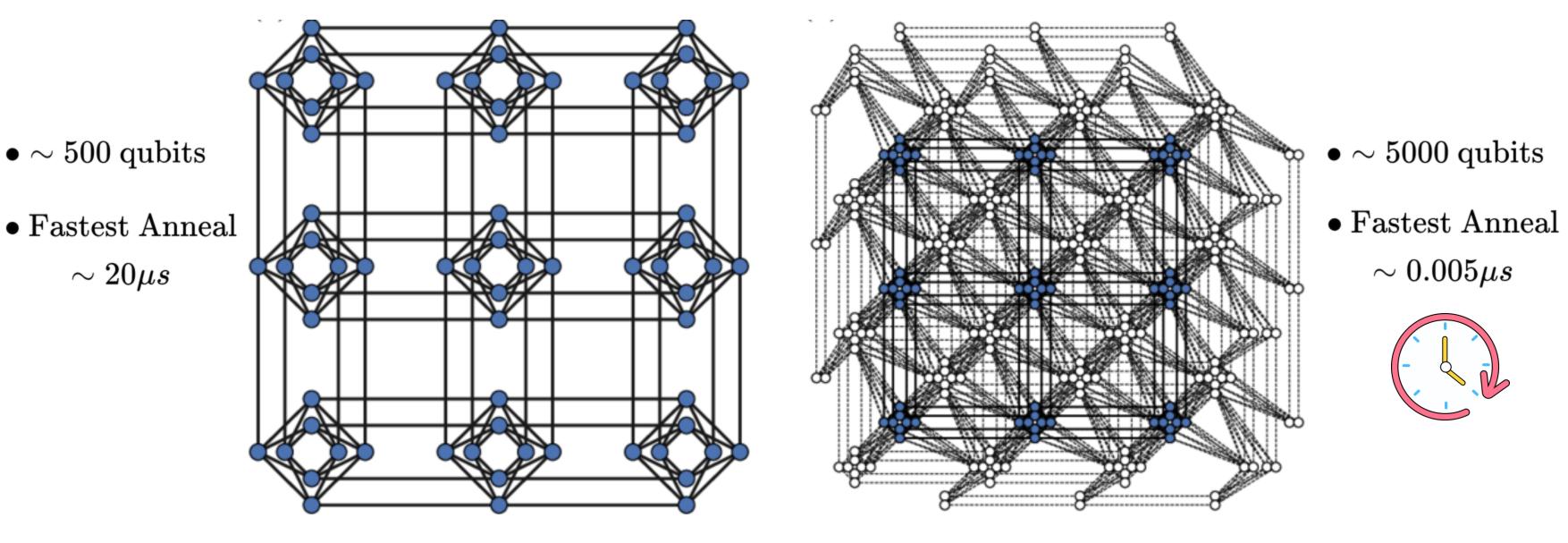
Quantum Information Processing. 20. 10.1007/s11128-021-03226-6 (2021).

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# Results...

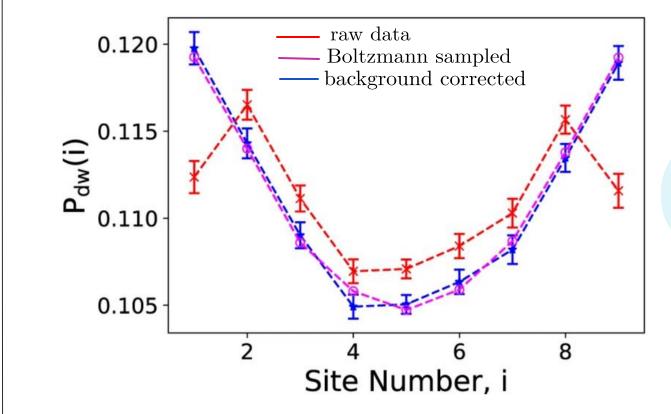
### **D-Wave Two**

### Default anneal regime

U-shape distribution due to random effective longitudinal fields.

End point susceptibility correction:

$$P_{dw}(i) \; 
ightarrow \; P_{dw}(i) \, igg( 1 + rac{B(t_{
m freeze})}{k_B T} \, \chi J(h-J) igg).$$



Standard Anneal time  $20 \mu s$ 

Domain wall distribution for a 10 qubit frustration chain on a Chimera graph, in npj Quantum Inf 8, 73(2022)

## Results...

### **D-Wave Two**

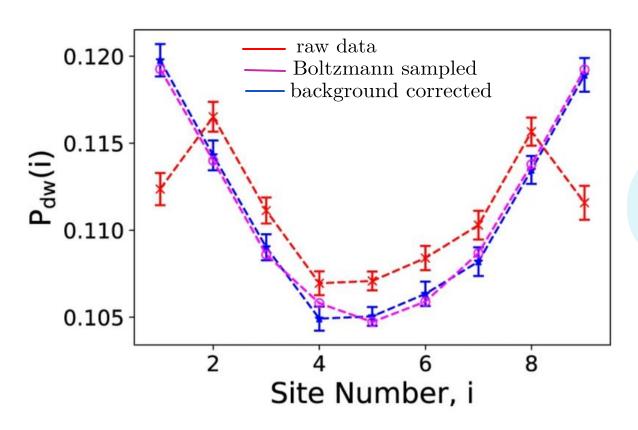
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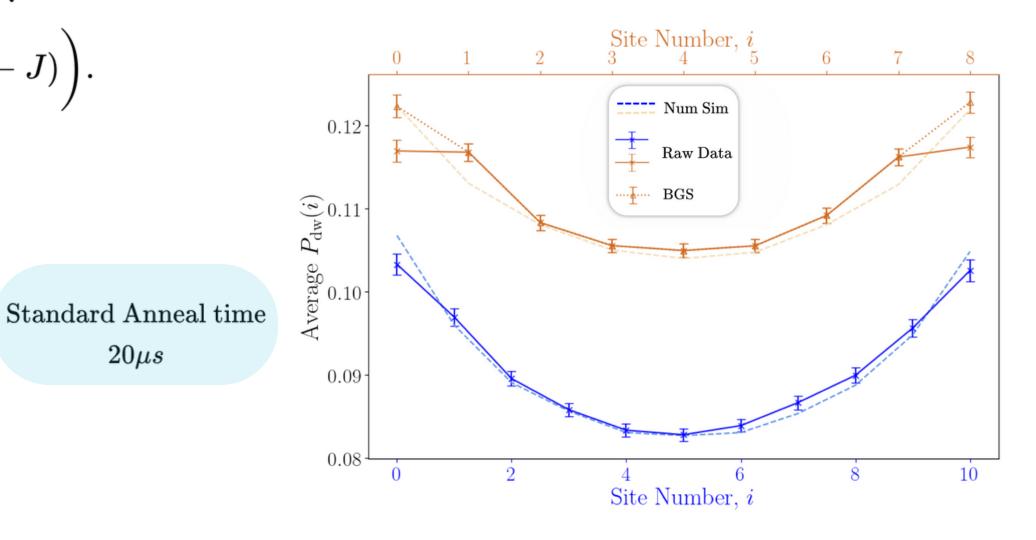
### Advantage 4.1

### Default anneal regime:

The distribution closely matches the original reporting.

### Fast anneal regime:

Lower probabilities and smoother edges.



Domain wall distribution for a 12 qubit frustration chain on a Pegasus graph, manuscript in prep

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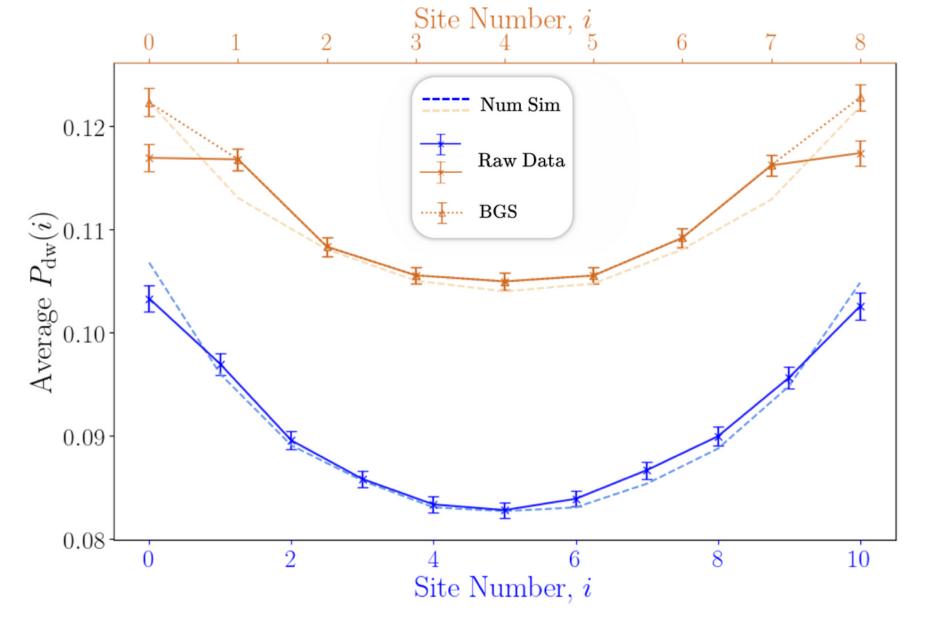
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# Coming back to single Qubit Control Errors

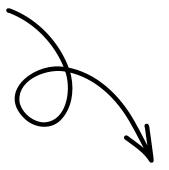


# Best Fit:

$$ullet \, P_{sim} = rac{e^{-eta \sum_{i=1}^n \epsilon_i}}{Z}$$

where  $\beta = \sigma_\epsilon/K_BT$ 

ullet  $P_{Exp}$ 



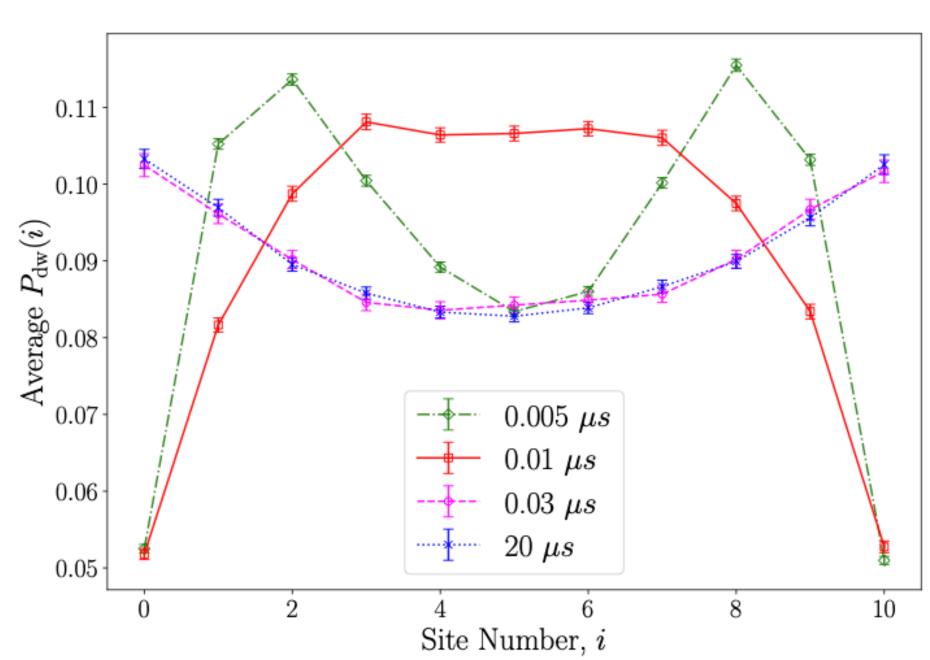
|        |         | L  | Time $(\mu s)$ | β     | $\sigma_{\zeta}$ |
|--------|---------|----|----------------|-------|------------------|
| Random | Default | 10 | 20             | 0.472 | 0.152            |
|        |         | 50 | 20             | 0.400 | 0.1284           |
|        | Fast    | 12 | 20             | 0.507 | 0.163            |
|        |         | 12 | 0.03           | 0.491 | 0.158            |

• Probability distribution in the fast anneal regime for a chain length of 12 with

chain length of 12 with 10,000 for different anneal times.

• For  $0.01~\mu s$ : particle in a box distribution f

• For later times the system thermalises , giving the U-shape



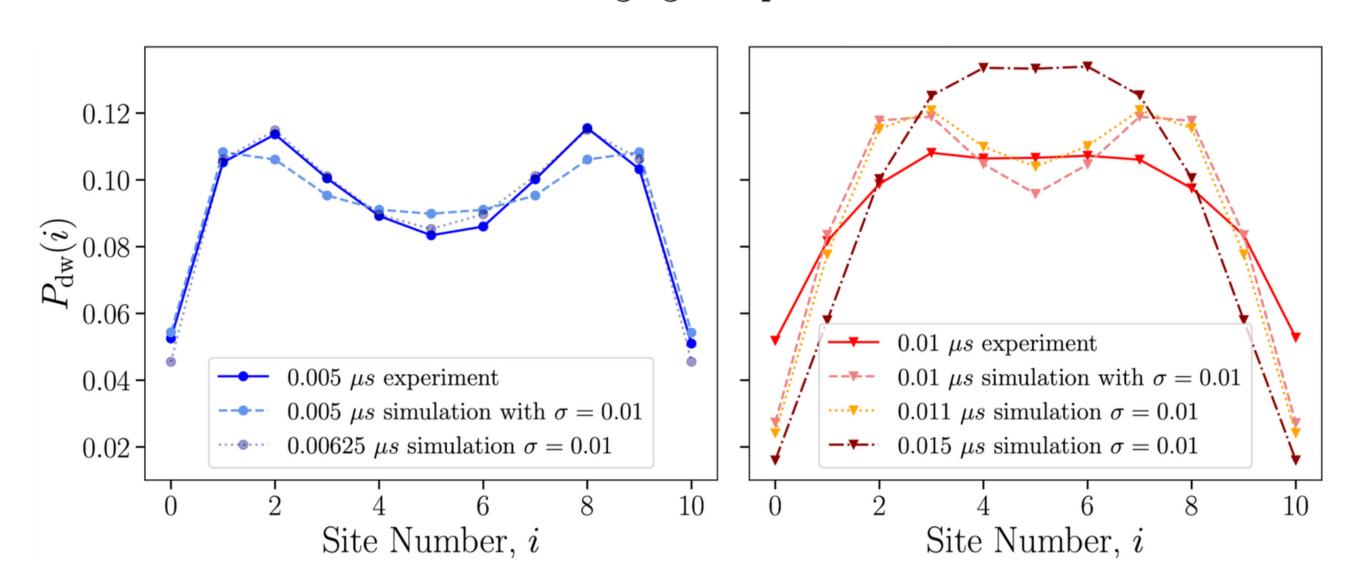
# Simulation I

Simulate coherent unitary dynamics in the presence of static disorder.

$$H_{prob} = J \sum_{i=1}^{N} -\sigma_i^z \sigma_{i+1}^z + h(\sigma_i^z - \sigma_{N+1}^z) + \sum_i \zeta_i \sigma_i^z$$

### Static noise simulation:

 $\zeta_i \sim \mathcal{N}(0, 0.01)$ , averaged over **10k realizations** to mimic the hardware's thermal averaging in experiments.

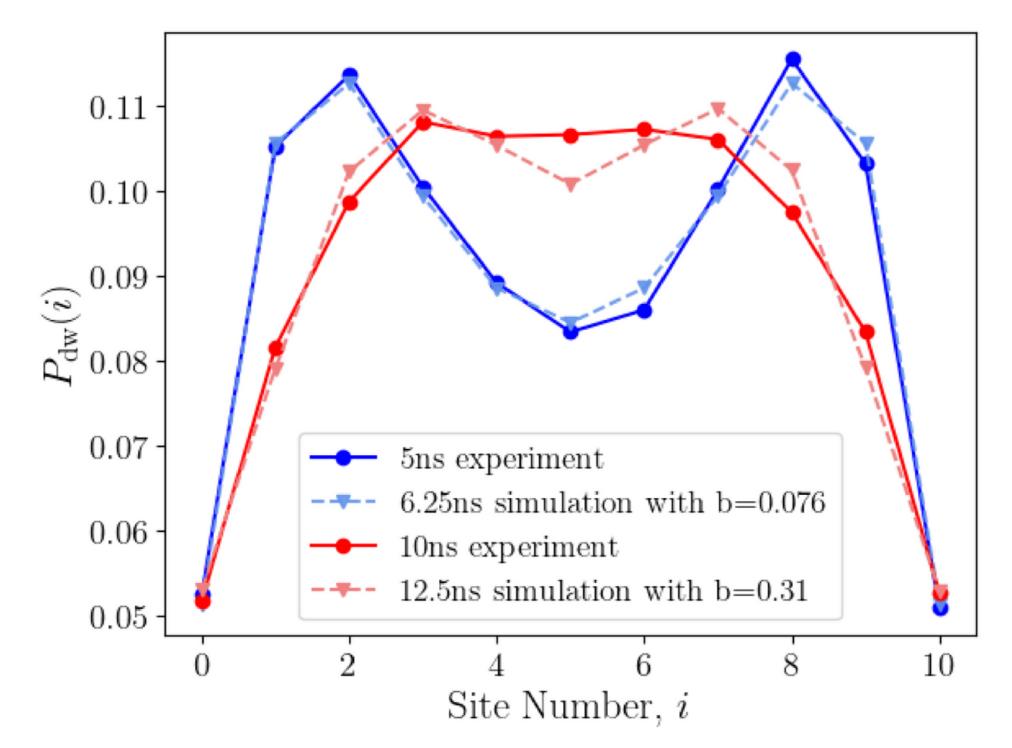


# Simulation II

• Model experimental data as a linear combination of quantum and thermal contributions

$$P_{Exp} = aP_{Quantum} + bP_{Thermal}$$

$$a + b = 1$$

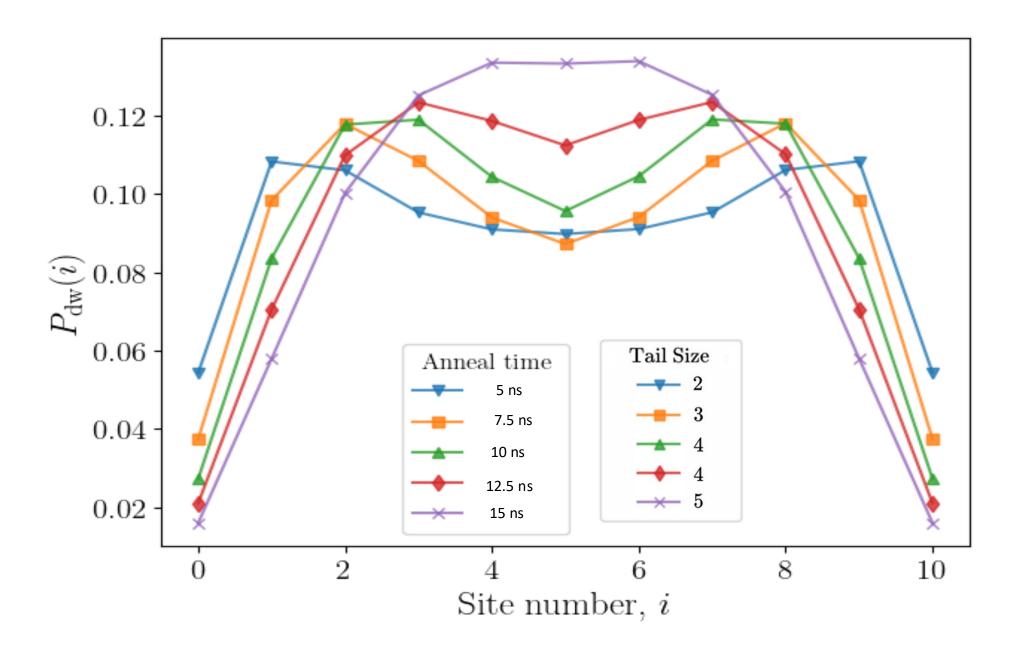


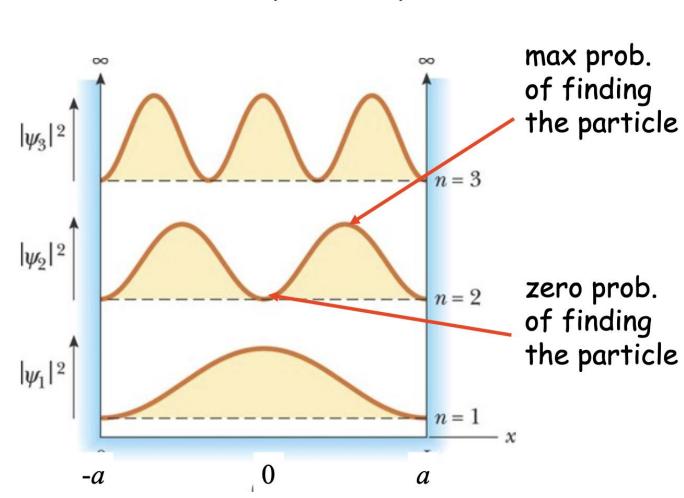
# What we set out to do...

Propose a general benchmarking framework in which the extent of the domain-wall tails serves as a robust metric of device performance,

capturing both coherence and noise levels.

Probability Density



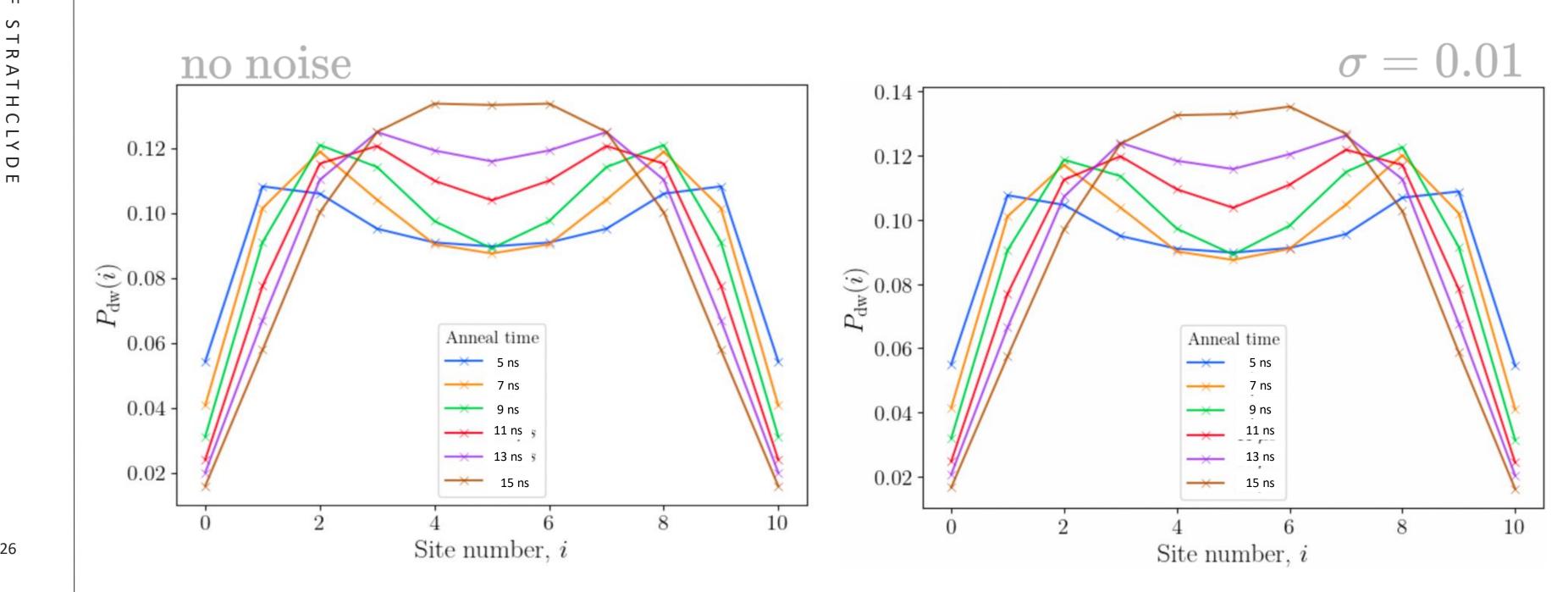


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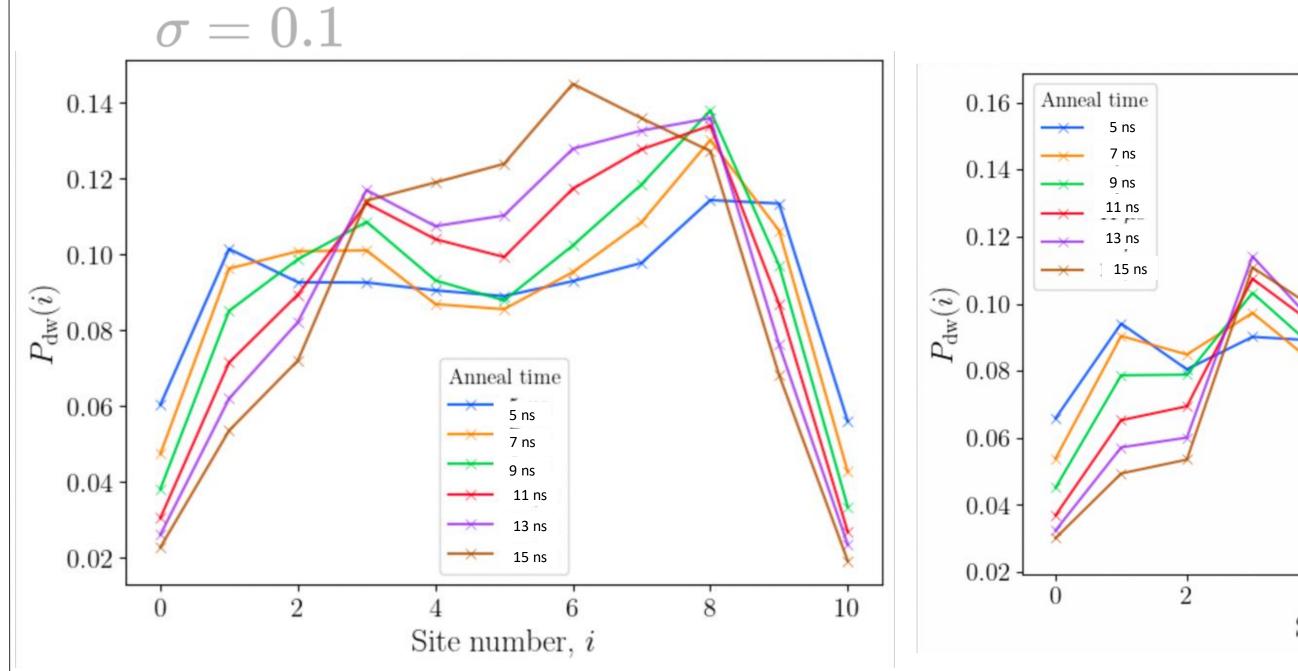
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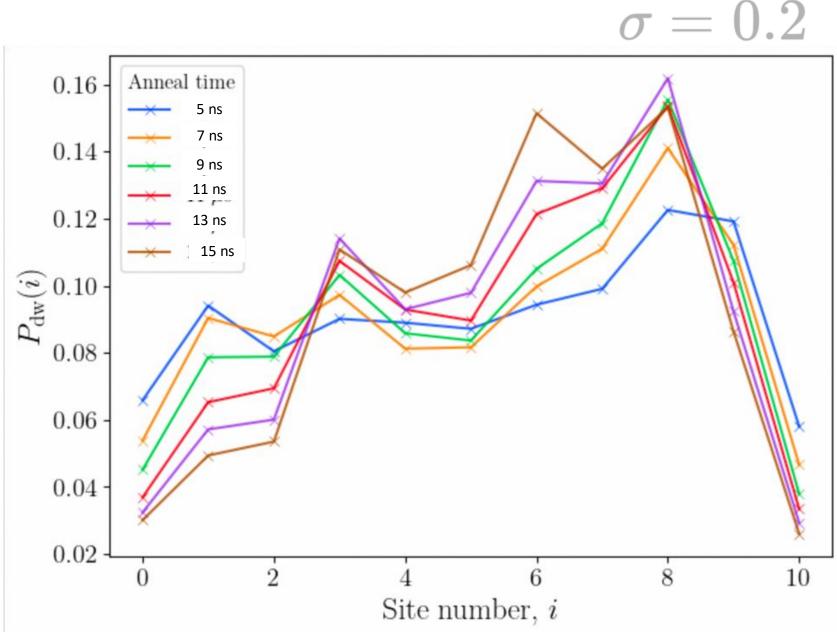
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# To conclude...

Quantum behaviour observed! Fast annealing  $(0.01 \ \mu s)$  on Advantage 4.1 produced the expected particle-in-a-box domain-wall distribution.

Reproduced results from [npj Quantum Inf. 8, 73 (2022)] and

extended the analysis to the Advantage 4.1 (Pegasus) QPU.

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Proposed a benchmarking framework where the extent of the domain-wall tails serves as a metric of device performance.



Tail captures both **coherence** and **noise levels**, providing a unified measure of analog device quality, and is **generalizable** to other platforms such as Rydberg-atom simulators.

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Dr.Léo Bourdet,
Simulation Engineer,
Microsoft

# Acknowledgements



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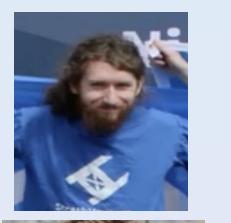
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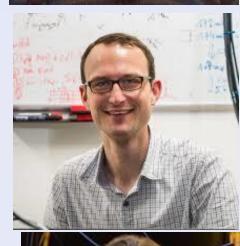
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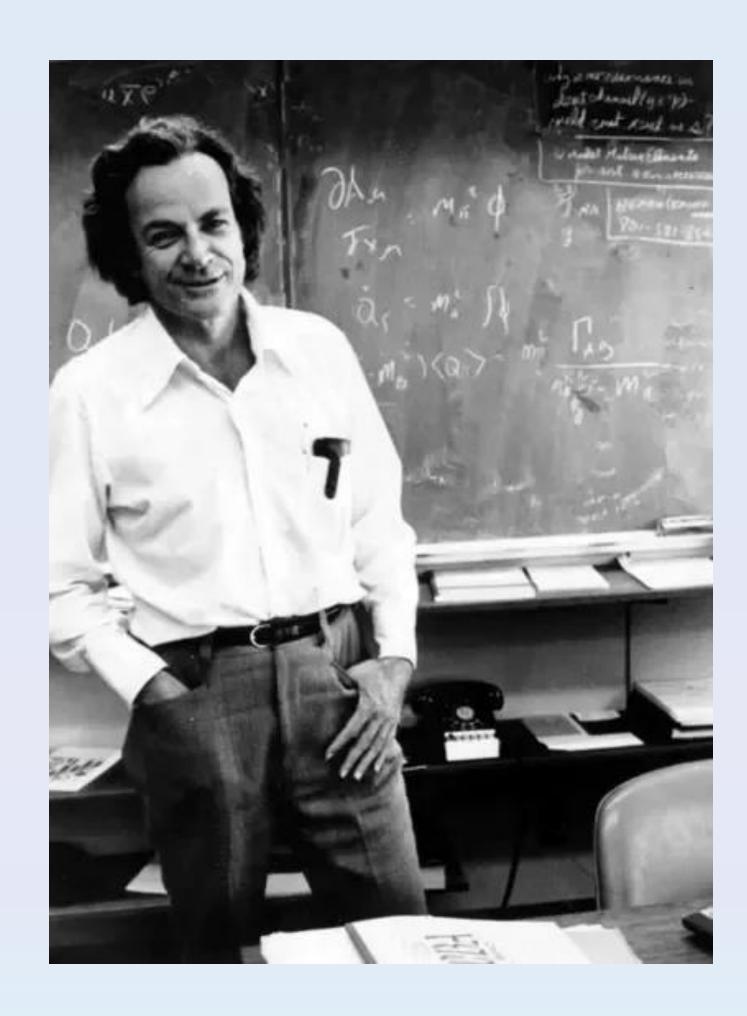
University of Strathclyde



Dr. Daniel Walker

Department of Physics,

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"The imagination of nature is far, far greater than the imagination of man."

— Richard P. Feynman